SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC) Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021
SEMESTER – III
14PAMCT3A07**&**PAM/CT/3A07– **Complex Analysis**

Total Duration : 3 hrs	**Total Mark : 75**
MCQ : 30 min	MCQ : 15
Descriptive : 2 Hrs. 30 Mins.	Descriptive : 60

Section B

Answer any ***Six*** questions (6 x 5 =30)

1. Let G be a region and suppose that f is a non constant analytic function on G. Prove that for any open set U in G, f (U) is open.

2. If f is a bounded entire function then prove that f is constant.

3. If f has an essential singularity at z = a then prove that for every $\delta > 0, \{f[ann(a; 0, \delta)]\}^{-} = \mathbb{C}$.

4. If f is analytic in a region G and a is a point in G with $|\boldsymbol{f}(\boldsymbol{a})| \geq |\boldsymbol{f}(\boldsymbol{z})|$ for all z in G then prove that f must be a constant function.

5. Let Re Zn >0 for all n ≥1. If $\prod_{\mathbf{n=1}}^{\infty} \mathbf{z_n}$ converges to a non zero number then prove that the series $\sum_{n=1}^{\infty} \boldsymbol{log\ z_n}$ converges.

6. If G = {z: Re z>0 } and fn(z) = $\int_{\mathbf{1/n}}^{\mathbf{n}} \mathbf{e^{-t}\, t^{z-1} dt}$ for n ≥1 and z in G then prove that each fnis analytic on G and the sequence is convergent in H(G).

7. If u: G → $\mathbb{C}$ is harmonic then prove that u is infinitely differentiable.

8. Let f be an entire function of finite order then show that f assumes each complex number with one possible exception.

Contd….

Section C

Part A

Answer any ***Two***questions (2x 10 =20)

9. State and Prove Cauchy's Integral Formula (First Version).
10. State and Prove Schwarz's Lemma.
11. For Re z > 1, then show that $\boldsymbol{\zeta(z)\Gamma(z)}=\int_{\mathbf{0}}^{\infty}(\boldsymbol{e^t-1})^{-\mathbf{1}}\,\boldsymbol{t^{z-1}}$dt.
12. If u : G → ℝ is a continuous function which has the MVP then prove that u is harmonic.

Part B

Compulsory Question (1 x 10 = 10)

13. If f is an entire function that omits two values, then prove that f is a constant.