SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATION APRIL/NOV - 2021
SEMESTER – I

17PAMCE1001 – Probability and Distributions

Total Duration : 3 hrs	**Total Mark : 75**
MCQ : 30 min	MCQ : 15
Descriptive : 2 Hrs. 30 Mins.	Descriptive : 60

Section B

Answer any ***SIX*** questions (6 x 5 =30)

1. Let X and Y be independent and identically distributed random variables, and let $P\{X = k\} = p_k > 0, k = 0,1,2,3.....$ If

$$P\{X = t / X + Y = t\} = P\{X = t-1 / X + Y = t\} = \frac{1}{t+1}, t \geq 0$$

then Prove that X and Y are geometric random variables.

2. Let (X_1,X_2) have uniform distribution on the triangle $\{0 \leq x_1 \leq x_2 \leq 1\}$; that is, (X_1,X_2) has joint density function $f(x_1,x_2) = \begin{cases} 2, 0 \leq x_1 \leq x_2 \leq 1 \\ 0, elsewhere. \end{cases}$ Find the density function of Y.

3. Prove that the bivariate binomial distribution generated by E_1 has joint probability mass function.

4. Let $X \sim F(m,n)$. Then, for k>0, integral, Prove that

(i) $E[X^k] = \left(\frac{n}{m}\right)^k \frac{\Gamma[k+(m/2)]\Gamma[(n/2)-k]}{\Gamma[(m/2)\Gamma(n/2)}$ for n>2k (ii) $E[X] = \frac{n}{n-2}, n > 2$

(iii) $\operatorname{var}(X) = \frac{n^2(2m+2n-4)}{m(n-2)^2(n-4)}, n > 4$

5. If $X_n \xrightarrow{r} X$ then prove that $E|X_n|^r \to E|X|^r$.

6. State and Prove Slutsky's theorem

7. Let F be any distribution function, and let X be a U[0,1] random variable. Then prove that there exists a function h such that h(X) has distribution function F, that is $P\{h(X) \leq x\} = F(x) \forall x \in (-\infty, \infty)$.

8. Prove that $X_n \xrightarrow{a.s} X \Rightarrow X_n \xrightarrow{P} X$.

Contd…

Section C

Part A

Answer any ***TWO*** questions (2 x 10 =20)

9. Define Gamma distribution with parameters α and β. Also compute Marginal Generating Function, Mean, Variance and $E[X^n]$ of the gamma distribution.

10. Let X_1, X_2 be independent random variables with common density given by

$$f(x) = \begin{cases} 1, if\ 0 < x < 1 \\ 0, Otherwise \end{cases}$$

Find Jacobian of the transformation, joint density of Y_1, Y_2 and the marginal probability density functions of Y_1 and Y_2

11. Explain Bivariate Poisson distribution.

12. Let $X_1, X_2, \ldots X_n$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables. Then prove that $\overline{X}$ and $(X_1 - \overline{X}, X_2 - \overline{X}, X_3 - \overline{X}, \ldots X_n - \overline{X})$ are independent. Also Prove that $(n-1)S^2/\sigma^2 is \chi^2(n-1)$.

Part B

Compulsory Question (1 x 10 = 10)

13. State and Prove Lindeberg - Levy Central Limit theorem.