#### SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS) (Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC) Chromepet, Chennai — 600 044. M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022 SEMESTER - II 20PAMCT2005 - Topology

Total Duration : 3 Hrs.

Total Marks : 60

## Section A

Answer any **SIX** questions  $(6 \times 5 = 30 \text{ Marks})$ 

- 1. State and prove Minkowski's Inequality.
- 2. Let X be a second countable space. Show that any open base for X has a countable subclass which is also an open base.
- 3. Show that every sequentially compact metric space is totally bounded.
- 4. Show that every compact Hausdorff space is normal.
- 5. Prove that any continuous image of a connected space is connected.
- 6. Let A be a subset of the topological space X; let D(A) be the set of all limit points of A. Then prove that  $\overline{A} = A \cup D(A)$
- 7. Prove that every sequentially compact metric space is compact.
- 8. Show that the product of any non-empty class of Hausdorff spaces is a Hausdorff space.

# Section B

## Part A

Answer any **TWO** questions  $(2 \times 10 = 20 \text{ Marks})$ 

- 9. Show that every closed and bounded subspace of the real line is compact.
- 10. State and prove Tychonoff's theorem.
- 11. Let X be a normal space, and let A and B be disjoint closed subspaces of X. Prove that there exists a continuous real function f defined on X, all of whose values lie in the closed unit interval [0,1], such that f(A)=0 and f(B)=1.
- 12. Prove that the product of any non empty class of connected space is connected.

## Part B

Compulsory question  $(1 \times 10 = 10 \text{ Marks})$ 

13. Let X and Y be metric spaces and f a mapping of X into Y. Prove that f is continuous if and only  $f^{-1}(G)$  is open in X whenever G is open in Y.

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