SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER - IV
20PAMCT4010 - Functional Analysis

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. 1. Let N and N' be normed linear space and T linear transformation of N into N'then the following conditions on T are equivalent to one another
   a. T is continuous
   b. T is continuous at origins in the sense that x$_n$→0 ⟹ T(x$_n$)→0
   c. there exists a real number K≥0 with the property that
   ‖T x‖ ≤ k ‖ X ‖ forever x∈N
   d. if S =x :‖x‖≤1 is the closed square in N then its image T(S)
   is bounded set in N'
2. State and prove uniform boundedness theorem
3. Prove that the adjoint operator T→T* on B(H) has the following properties
   a. (T$_1$+ T$_2$)*= T$_1$*+T$_2$*
   b. (αT)*=$\bar{\alpha}$T*
   c. (T$_1$T$_2$)*= T$_2$*T$_1$*
   d. T** = T
   e. ‖T*‖ = ‖T‖
   f. ‖T*T‖= ‖T‖^2= ‖TT*‖
4. In Banach algebra prove that The mapping x→ x^{-1} of G into G is continuous and it is therefore a homomorphism of G onto itself.
5. Prove that if f$_1$ and f$_2$ are multiplicative functionals on A with the small null space then f_1=f_2
6. State and prove open mapping theorem
7. State and prove Bessel's inequality
8. Let M be a closed linear subspace of a normed linear space N.If the norm of a coset x + M in the quotient space N/M is defined by
   ‖ x+M‖ = inf {‖ x+M‖ : m ∈ M} then prove that N/M is a normed linear space. Further if N is a Banach space then so is N/M

Contd...

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. If M and N are closed linear subspace of a Hilbert space H such that $M \perp N$, then the linear subspace $M + N$ is also closed also prove that M is close linear subspace of a Hilbert space H then $H = M \oplus M^{\perp}$
10. If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other then prove that $N_1 +$ N_2 and $N_1 N_2$ are normal also prove that an operator T on H is normal $\iff$ $\|T^*x\| = \|Tx\|$ for every x and $\|$ T2 $\| = \|$ T $\|^2$
11. Prove that following in Banach Algebra
   a. If r is an element of R then 1 –r left regular
   b. If r is an element of R then 1 –r regular
   c. If r is an element of R then 1 –x r regular for every x
   d. If r is an element of Awith the property that 1 –xrregular for every x then r is in R
12. State and prove Gelfand- Neumark theorem .

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. Let M be a linear subspace of a normed linear space N and let f be a functional defined on M. if x_0 is a vector not in M and if $M_0 = M + [x_0]$ is the linear subspace spanned by M and x_0, then show that f can be extended to a functional f_0 defined on M_0 such that $\| f_0 \| = \| f \|$

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