SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER - IV
20PAMCT4011 - Differential Geometry and Tensor Calculus

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Obtain the equations of the circular helix $r = (a\cos u, a\sin u, bu)$, $-\infty < u < \infty$ where $a > 0$, referred to s as parameter, and show that the length of one complete turn of the helix is $2\pi c$, where $c = \sqrt{a^2 + b^2}$.
2. On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z =$ constant.
3. Prove that, on the general surface, a necessary and sufficient condition that the curve v=c be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when v=c, for all values of u.
4. Find the cylindrical coordinates of the point whose rectangular Cartesian coordinates are (6, 8, 5).
5. Determine the metric tensor and the conjugate metric tensor in cylindrical coordinates.
6. If A^i and B_i are a contravariant and a covariant vector respectively, then the sum $A^i B_i$ is an invariant.
7. Show that the length of the common perpendicular d of the tangents at two near points distance s apart is approximately given by $d = \frac{kts^3}{12}$.
8. A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1, a'x^2 + b'y^2 + c'z^2 = 1$.
10. A helicoid is generated by the screw motion of a straight line which meets the axis at an angle a. Find the orthogonal trajectories of the generators. Find also the metric of the surface referred to the generators and their orthogonal trajectories as parametric curves.

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11. Find the geodesics on a surface of revolution.
12. A covariant vector has components 2x, y^2 - z, z^2 in rectangular coordinates. Determine its covariant components in cylindrical coordinates.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. Prove that the covariant derivative of the fundamental tensors g_{ij}, g^{ij} and δ^i_j vanish identically.

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