SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai — 600 044.

M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022

SEMESTER - I

20PAMET1001 - Probability and Distributions

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let X be a nonnegative integer-valued RV satisfying P{X>m+1|X>m}=PX $\geq$ 1 for any nonnegative integer m. Indentify X must have a geometric distribution
2. Let X and Y be independent RVs and f and g be Borel-measurable functions. Predict f(X) and g(Y) are also independent.
3. Interpret Cov (X, Y) $= \rho\sigma_1\sigma_2$.
4. Let X $\sim$ F(m, n).Show, for k $>$ 0, integral, $EX^k = \left(\frac{n}{m}\right)^k \frac{\Gamma[k+(m/2)]\Gamma[(n/2)-k]}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}$ for n>2k and EX= n / n - 2, n $>$ 2.
5. Let $X_n \xrightarrow{P} X$, and g be a continuous function defined on $\mathcal{R}$. Apply $g(X_n) \xrightarrow{P} g(X)$ as n $\to \infty$.
6. Solve the RV $X_{(k)}$ has a beta distribution with parameters α = k and β = n – k + 1.
7. Compute X and Y are independent RVs if and only if M (t_1, t_2) = M (t_1, 0) M (0, t_2) for all t_1, t_2 in $\mathcal{R}$.
8. Let {X_n, Y_n}, n = 1, 2, ..., be sequence of RVs . Determine $|X_n - Y_n| \xrightarrow{P} 0$ and $Y_n \xrightarrow{L} Y \Rightarrow X_n \xrightarrow{L} Y$.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Let F be any DF, and let X be a U [0, 1] RV. Describe there exists function h such that h(X) has DF, that is P{ h(X) $\leq$ x} = F(x) for all x $\in$ (-∞,∞).
10. Let (X, Y) be jointly distributed with density function

    $$f(x,y)= \begin{cases} x+y & 0<x<1, 0<y<1 \\ 0 & otherwise \end{cases}$$

    Examine the correlation coefficient ρ of X and Y.

Contd...

11. Let (X_1, X_2, ..., X_n) be an n-dimensional RV with a normal distribution. Let Y_1, Y_2, ..., Y_k, k $\leq$ n, be linear functions of Xj (j = 1, 2, ..., n). Classify (Y_1, Y_2, ..., Y_k) also has a multivariate normal distribution.

12. Let X_1, X_2, ..., X_n be i.i.d N(μ,σ^2) RVs. Deduce X_1 - $\bar{X}$,X_2 - $\bar{X}$,...,X_n -$\bar{X}$ are independent.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. Justify Lindeberg-Levy Central Limit Theorem.

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