SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.

M.Sc. END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER – II
20PAMET2002 – Mathematical Statistics

Total Duration : 3 hrs   Total Marks : 60

Section A

Answer any ***SIX*** questions   (6 x 5 =30)

1. If T_n is a sequence of estimators such that $ET_n \to \psi(\theta)$ and $\boldsymbol{var\,(T_n) \to 0}$ as $\boldsymbol{n \to \infty}$, indicate $\boldsymbol{T_n}$ is consistent for $\boldsymbol{\psi(\theta)}$.
2. Let $\boldsymbol{X_1, X_2, \dots, X_n}$ be a sample from $\boldsymbol{G\left(r, \frac{1}{\beta}\right); \beta > 0}$ and r > 0 are both unknown. Solve the likelihood function is

$$\boldsymbol{L(\beta, r;\ x_1, x_2, \dots, x_n) = \begin{cases} \dfrac{\beta^{nr}}{[\Gamma(r)]^n} \prod_{i=1}^{n} x_i^{r-1} exp\,(-\beta \sum_{i=1}^{n} x_i), x_i \geq 0 \\ 0 \qquad\qquad\qquad otherwise \end{cases}}$$

3. Let T(x) be maximal invariant with respect to $\mathscr{G}$. Describe φ is invariant under $\mathscr{G}$ if and only if φ is a function of T.

4. Identify the following data were obtained from a table of random numbers of normal distribution with mean 0 and variance 1.

0.464	0.137	2.455	– 0.323	– 0.068
0.906	–0.513	–0.525	0.595	0.881
–0.482	1.678	–0.057	–1.229	–0.486
–1.787	–0.261	1.237	1.046	–0.508

We want to test the null hypotheses that the DE F from which the data.

Contd..

5. Apply three sections of the same elementary statistics course were taught by three instructors, I, II and III. The final grades of students were recorded as follows:

I	II	III
95	88	68
33	78	79
48	91	91
76	51	71
89	85	87
82	77	68
60	31	79
77	62	16
	96	35
	81	

Let us test the hypothesis that the average grades given by the three instructors are the same at level $\alpha = 0.05$.

6. If S(X) is a complete sufficient statistic for θ, then classify any ancillary statistic A(X) is independent of S.

7. Let $\mathbf{X_1}, \mathbf{X_2}, \dots, \mathbf{X_n}$ be a sample from N (μ, σ2), where σ2 is known. Interpret $\overline{\mathbf{X}}$ is sufficient for μ and take $\mathbf{T_\mu(X)} = \frac{\overline{\mathbf{X}}-\mu}{\sigma/\sqrt{\mathbf{n}}}$.

8. Justify Nine determinations of copper in a certain solution yielded a sample mean of 8.3 percent with a standard deviation of 0.025 percent. Let μ be the mean of the population of such determinations Let us test $H_0 : \mu = 8.42$ against $H_1 : \mu < 8.42$ at level $\alpha = 0.05$.

Section B

Part A

Answer any ***TWO*** questions (2x 10 =20)

9. Apply Rao-Blackwell theorem.

10. (i) Let T be a sufficient statistic for the family of PDFs $\{f_\theta : \theta \epsilon \Theta\}$ If a unique MLE of θ exists, it is a function of T. If a MLE of θ exists but is not unique, one can distinguish a MLE that is a function of T.

(ii) Differentiate that the regularity conditions of the FCR inequality are satisfied and θ belongs to an open interval on the real line. If an estimator $\hat{\boldsymbol{\theta}}$ of θ attains the FCR lower bound for the variance, the likelihood equation has a unique solution $\hat{\boldsymbol{\theta}}$ that maximizes the likelihood.

11. Consider the problem of testing μ = μ0 against μ ≠ μ0 in sampling from N (μ, σ2), where both μ and σ^2 are unknown. Compute G.L.R test.

12. Evaluate One-Way Analysis of Variance.

Part B

Compulsory Question (1 x 10 = 10)

13. Organize Neyman-Pearson Fundamental Lemma.

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