SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER - IV
20PAMET4004 - Calculus of Variations and Integral Equations.

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that the functional I_1 [y(x)] = $\int_a^b$[y'(x)+y(x)] dx is linear in the class . C$^1[a, b]$, but the functional I_2 [y(x)] = $\int_a^b$[p(x) [y'(x)]^2 + q(x) y^2 (x)] dx is non-linear.
2. Find the extremals of the functional I=$\int_0^{\pi/2}(y'^2 + z'^2 + 2yz)$ dz subject to the boundary conditions y(0)=0, y(π/2)=1, z(0)=0, z(π/2)=-1.
3. Test for extremum of the functional I [y(x)]=$\int_0^{\pi/2}$(y'^{2} - y^2)dx, y(0) = 0, y(π/2)=1.
4. Show that the function y(x)=(1+x^2)$^{-3/2}$ is a solution of the Voterra integral equation y(x)=$\frac{1}{1+x^2}$ - $\int_0^x \frac{t}{1+x^2}$ y(t) dt.
5. Using Fredholm determinant, find the resolvent kernel of the integral equation y(x)=f(x)+$\lambda\int_0^1$ xe^t y(t)dt, ($\lambda\neq$1) and hence solve it.
6. Using Fredholm determinants, find the resolvent kernel, when K(x,t)=xe^t, a=0, b=1.
7. State and prove Hilbert theorem.
8. Solve the integral equation f(x)=$\int_a^x \frac{y(t)dt}{(\cos t - \cos x)^{1/2}}$ 0$\leq a<$x<b$\leq\pi$.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that a differential functional has the unique differential.
10. Find the geodesics of the sphere.
11. Solve y(x)=$f(x)$+$\lambda\int_0^1$(1-3xt)y(t)dt.

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12. Solve y(x)=1+$\int_0^1$(1-3xt)y(t)dt.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. Obtain the solution of the Cauchy-Type singular integral equation.

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