SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER - III
14PAMCT3A07 - Complex Analysis

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let G be a region and suppose that f is a non-constant analytic on G, then prove that for any open set U in G, F(U) is open.
2. If f:G$\to$C is an analytic function and γ is a rectifiable curve in G such that $\gamma \sim 0$, then prove that $\int_\gamma f = 0$.
3. State and prove Casorati- Weierstrass theorem.
4. Show that $\int_0^\infty \frac{\sin x}{x}$ dx $= \frac{\pi}{2}$.
5. If Re z > 1, then prove that $\varepsilon(z) = \prod_{n=1}^{\infty} \left(\frac{1}{1-p_n^{-z}}\right)$ where $\{P_n\}$ is a sequence of prime numbers.
6. State and prove Harnack's inequality.
7. If u:G$\to$R is a continuous function which has the mean value property then prove that u is a harmonic.
8. If f is an entire function that omits two values, then prove that f is a constant.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Let γ be a rectifiable curve and suppose $\varphi$is a function defined and continuous on $\{\gamma\}$. For m$\geq$1,Let $F_m(z) = \int_\gamma \varphi(w)(w-z)^{-n}$ dw for z$\notin\{\gamma\}$. Then prove that each F_m is analytic on C-$\{\gamma\}$ and $F'_m(z) = mF_{m+1}(z)$.
10. State and prove Argument principle.
11. State and prove Riemann mapping theorem.
12. State and prove Harnack's Theorem.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Jensen's formula.

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