

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.

M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022

SEMESTER - IV

17PAMCE4A05 - Operations Research

Total Duration : 3 Hrs.

Total Marks : 60

Section A

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Write the dual of the following LP Problem

$$\text{Minimize } z = 3x_1 - 2x_2 + 4x_3$$

Subjects to the constraints

$$3x_1 + 5x_2 + 4x_3 \geq 7$$

$$6x_1 + x_2 + 3x_3 \geq 4$$

$$7x_1 - 2x_2 + 5x_3 \leq 10$$

$$x_1 - 2x_2 + 5x_3 \geq 3$$

$$4x_1 + 7x_2 - 2x_3 \geq 2$$

$$\text{And } x_1, x_2, x_3 \geq 0$$

2. Determine the value of u_1, u_2, u_3 so as to maximize $Z = u_1 \cdot u_2 \cdot u_3$

Subject to the constraints

$$u_1 + u_2 + u_3 = 10$$

$$\text{And } u_1, u_2, u_3 \geq 0$$

3. A commodity is to be supplied at a constant rate of 200 units per day. Supplies of any amount can be obtained at any required time, but each ordering costs Rs 50; cost of holding the commodity in inventory is Rs 2 per unit per day while the delay in the supply of the item induces a penalty of Rs 10 per unit per day. Find the optimal policy (Q, t) , where t is the reorder cycle period and Q is the inventory after reorder. What would be the best policy, if the penalty cost becomes infinite?
4. Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone call is assumed to be distributed exponentially, with mean 3 minutes.
- (a) What is the probability that a person arriving at the booth will have to wait?
- (b) What is the average length of the queue that forms from time to time?
- (c) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?
5. A trader receives x units of an item at the beginning of each month. The cost of carrying x units per month is given by

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$$C(x) = \frac{c_1 x^2}{2n} + \frac{c_2(20n-x)^2}{n} = 0$$

Where C_1 = cost per day of carrying a unit of item in stock (= Rs 10)

C_2 = cost per day of shortage of a unit of item (= Rs 150)

n = number of units of item to be supplied per day (30)

Determine the order quantity x that minimizes the cost of inventory.

6. Use the Wolfe's method to solve the quadratic programming problem

$$\text{Maximize } Z = 2x_1 + x_2 - x_1^2$$

subject to the constraints

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

$$\text{And } x_1, x_2 \geq 0$$

7. The production department for a company requires 3,600 kg of raw material for manufacturing a particular item per year. It has been estimated that the cost of placing an order is Rs 36 and the cost of carrying inventory is 25 per cent of the investment in the inventories. The price is Rs 10 per kg. The purchase manager wishes to determine an ordering policy for raw material.
8. At a certain airport it takes exactly 5 minutes to land an aeroplane, once it is given the signal to land. Although incoming planes have scheduled arrival times the wide variability in arrival times produces an effect which makes the incoming planes appear to arrive in a Poisson fashion at an average rate of 6 per hour. This produces occasional stockups at the airport which can be dangerous and costly. Under these circumstances, how much time will a pilot expect to spend circling the field waiting to land?

Section B

Part A

Answer any **TWO** questions ($2 \times 10 = 20$ Marks)

9. Use dual simplex method to solve the LP problem

$$\text{Maximize } z = 3x_1 - 2x_2$$

Subject to the constraints

$$x_1 + x_2 \geq 1$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \geq 0$$

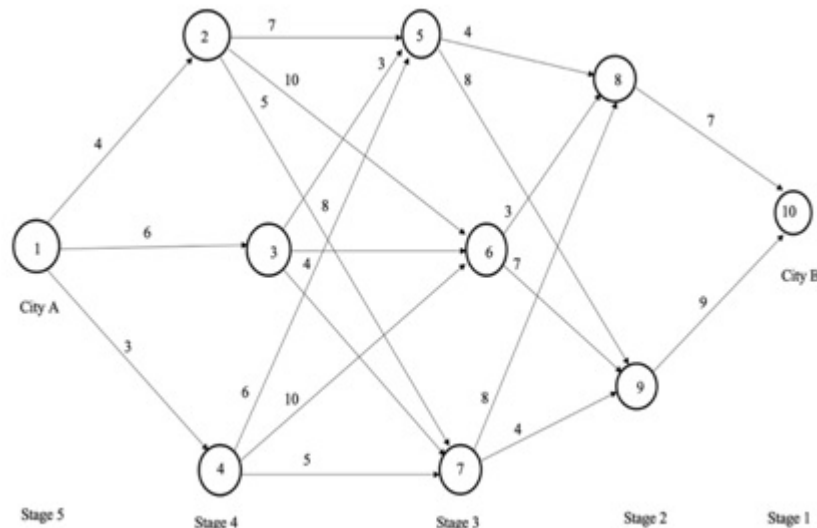
$$x_2 \leq 3$$

$$\text{and } x_1, x_2 \geq 0$$

10. A salesman located in a city A decided to travel to city B. He knew the distances of alternative routes from city A to city B. He then drew a highway network map as shown.

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The city of origin, A, is city 1. The destination city B, is city 10. Other cities through which the salesman will have to pass through are numbered 2 to 9. The arrow representing routes between cities and distances in kilometres are indicated on each route. The salesman's problem is to find the shortest route that covers all the selected cities from A to B.



11. A dealer supplies you the following information with regard to a product dealt in by him: Annual demand = 10,000 units; Ordering cost = Rs 10 per order; Price = Rs 20 per unit Inventory carrying cost = 20 per cent of the value of inventory per year. The dealer is considering the possibility of allowing some backorder (stockout) to occur. He has estimated that the annual cost of backordering will be 25 per cent of the value of inventory.
- (a) What should be the optimum number of units of the product he should buy in one lot?
- (b) What quantity of the product should be allowed to be backordered, if any?
- (c) What would be the maximum quantity of inventory at any time of the year?
- (d) Would you recommend to allow backordering? If so, what would be the annual cost saving by adopting the policy of backordering.
12. Derive the EOQ model with constant demand and fixed order cycle time with gradual supply and shortages allowed.

Part B

Compulsory question (1 × 10 = 10 Marks)

13. Each unit of an item costs a company Rs 40. Annual holding costs are 18 per cent of unit cost for interest charges, 1 per cent for insurance, 2 per cent allowances

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for obsolescence, Rs 2 for building overheads, Rs 1.50 for damage and loss, and Rs 4 miscellaneous costs. Annual demand for the item is constant at 1,000 units and each order costs Rs 100 to place.

(a) Calculate EOQ and the total costs associated with stocking the item.

(b) If the supplier of the item will only deliver batches of 250 units, how are the stock holding costs affected?

(c) If the supplier relaxes his order size requirement, but the company has limited warehouse space and can stock a maximum of 100 units at any time, what would be the optimal ordering policy and associated costs?
