SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai — 600 044.

M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022

SEMESTER - IV

17PAMCT4A11 - Differential Geometry and Tensor Calculus

Total Duration : 3 Hrs.   **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$.

2. A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

3. Prove that the curves of the family $v^3/u^2 = $ constant are geodesics on a surface with metric $v^2 \, du^2 - 2uv \, dudv + 2u^2 \, dv^2$ ($u > 0$, $v > 0$).

4. Prove that, if θ is the angle at the point (u, v) between the two directions given by the quadratic differential equation $Pdu^2 + 2Qdudv + Rdv^2 = 0$,
   then $\tan \theta = \dfrac{2H(Q^2 - PR)}{ER - 2FQ + GP}$.

5. Prove that if (λ, μ) is the geodesic curvaturevector, then $K_g = \dfrac{-H\lambda}{Fu' + Gv'} = \dfrac{H\mu}{Eu' + Fv'}$.

6. The sum (or difference) of two tensors which have the same number of covariant and the same number of Contravariant indices is again a tensor of the same type and rank as the given tensors.

7. If a_{ij} is a skew symmetric tensor and A^i is a covariant vector then prove that $a_{ij} A^i A^j = 0$.

8. Show that $\dfrac{\partial g_{ij}}{\partial x^k}$ - $\dfrac{\partial g_{ik}}{\partial x^i}$ = [jk,i]-[ij,k].

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is$EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$, for all values of u.

10. State and prove Gauss - Bonnet theorem .

Contd...

11. If a transformation of coordinates T possesses an inverse T^{-1} and if J and K are the Jacobians of T and T^{-1} respectively then prove that JK=1 .

12. State and prove Ricci's theorem.

Part B

Compulsory question $(1 \times 10 = 10$ Marks$)$

13. A helicoid is generated by the screw motion of a straight line which meets the axis at an angle α. Find the orthogonal trajectories of the generators. Find also the metric of the surface referred to the generators and their orthogonal trajectories as parametric curves.

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