SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER - I
20PAMCT1001 - Algebra – I

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If p is a prime number and p | o(G), describe G has an element of order p.
2. Suppose that G is the internal direct product of N_1, N_2,..., N_n, apply for i $\neq$ j, Ni $\cap$ Nj = (e), and if a $\in$ Ni, b $\in$ Nj then ab = ba.
3. Explain Jacobson Lemma.
4. Let F be a finite field with q elements and suppose that F $\subset$ K where K is also a finite field. Show K has q^n elements where n = [K: F].
5. Illustrate the adjoint in Q satisfies
(i) $x^{**} = x$, (ii) $(\delta x + \gamma y)^* = \delta x^* + \gamma y^*$.
6. Compute $n(k) = 1 + p + ... + p^{k-1}$.
7. Interpret G is solvable if and only if $G^{(k)} = (e)$ for some integer k.
8. If N is normal and AN = NA, determine $AN^* = N^*A$.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Classify the number of p-sylow subgroups in G, for a given prime, is of the form 1 + kp.
10. Organize Sn is not solvable for n $\geq$ 5.
11. If F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F, apply we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.
12. Appraise Left-Division Algorithm.

Contd...

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. If $T \in A(V)$ then $T^* \in A(V)$. Examine
    (i) $(T^*)^* = T$,
    (ii) $(S + T)^* = S^* + T^*$,
    (iii) $(\lambda S)^* = \bar{\lambda} S^*$,
    (iv) $(ST)^* = T^* S^*$
    for all $S, T \in A(V)$ and all $\lambda \in F$.

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