SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER - I
20PAMCT1002 - Real Analysis

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Indicate for any sequence of sets $\{E_i\}$,$m^* (\cup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^* (E_i)$.
2. Apply Lebesgue's Dominated Convergence Theorem.
3. Let α be monotonically increasing on [a, b]. Suppose $f_n \in \mathfrak{R}(\alpha)$ on [a, b], for n = 1, 2, 3,..., and suppose $f_n \to f$ uniformly on [a, b]. Describe $f \in \mathfrak{R}(\alpha)$ on [a, b] and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.
4. If X is a complete metric space, and if ϕ is a contraction of X into X, interpret there exists one and only one x $\in$ X such that ϕ(x) = x.
5. Suppose a_0, a_1,...,a_n are complex numbers, n $\geq$ 1, an $\neq$ 0, P(z)=$\sum_0^n a_k z^k$. Compute P(z) =0 for some complex number z.
6. Let $\{$fn$\}$ be a sequence of measurable functions defined on the same measurable set. Classify
   (i) sup 1 $\leq$ i $\leq$ n f_i is measurable for each n,
   (ii) inf 1 $\leq$ i $\leq$ n f_i is measurable for each n,
   (iii) sup f_n is measurable,
   (iv) inf f_n is measurable,
7. If f is Riemann integrable and bounded over the finite interval [a, b], show f is integrable and R $\int_a^b f dx$= $\int_a^b f dx$.
8. Suppose E and f are as in differentiable at x, x $\in$ E, with A = A_1 and with A = A_2. Examine A_1 =A_2.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Describe the class M is a σ-algebra.
10. Diagnose Fatou's Lemma.

Contd...

11. Suppose f maps an open set E$\subset$R^n into R^m. Apply f$\in\wp$' (E) if and only if the partial derivatives D$_j$ f$_i$ exist and are continuous on for $1 \leq$ i $\leq$ m, $1 \leq$ j $\leq$ n.

12. Appraise Parseval's Theorem.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. Justify the Stone-Weierstrass Theorem.

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