SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS APRIL - 2022
SEMESTER - II
20PAMCT2004 - Algebra II

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that the elements in K which are algebraic over F form a subfield of K.
2. Define a splitting field and compute the splitting field of $x^3 - 1$.
3. Define G(K,F) and compute G(Q($\sqrt[3]{2}$,Q) .
4. Show that the matrix $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent and find its invariants.
5. If V = $V_1 \oplus V_2 \oplus \ldots \oplus V_k$ where each V_i is invariant under T and if p_i(x) is the minimal polynomial over F of T_i, the linear transformation induced by T on V_i,then show that the minimal polynomial of T over F is the LCM of p_1(x),p_2(x),. . . ,p_k(x).
6. Compute the Galois group of x^3-2 over Q.
7. Define similarity between two linear transformations in A(V) and show that this similarity relation in A(V) is an equivalence relation.
8. Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. Show that an element $a \in$K is algebraic over F if and only if F(a) is a finite extension of F.
10. If F is of characteristic zero, and if a,b,are algebraic over F, then show that there exists an element $C \in$F(a,b)such that F(a,b) = F(c).
11. Define nilpotent linear transformation. Also prove that the characteristic roots of a nilpotent transformation are all zero.
12. Prove that every linear transformation T$\in A_F$(V) satisfies its characteristic polynomial. Also show that every characteristic roots of T is the root of p_T(x).

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Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. Prove that the field K is a normal extension of F if and only if K is a splitting field of some polynomial over F

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