SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS APRIL-2022
SEMESTER - V
17UMACT5A10 & UMA/CT/5A10 - **Real Analysis**

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that countable union of countable set is countable.
2. If (S_n) is a Cauchy sequence of real numbers, then prove that (S_n) is convergent.
3. Let(M,ρ) be metric space and let 'a' be a point in M.Let f and g be a real valued functions, whose domains are subset of M.If $\underset{x \to a}{Lt} f(x) =$ L and $\underset{x \to a}{Lt}\ g(x) =$ N then prove that $\lim_{x \to a}[f(x)g(x)] = LN$
4. If the subset A of the metric space (M,ρ) is totally bounded, then prove that A is bounded.
5. State and prove Rolle's theorem.
6. State and prove Ratio test for series.
7. State and prove Picard fixed point theorem.
8. Prove that every absolutely convergent series is convergent

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that a non decreasing sequence which is bounded above is convergent. Hence, deduce that the sequence $\left[(1 + \frac{1}{n})^n\right]_{n=1}^{\infty}$ is convergent.
10. If $(a_n)_{n=1}^{\infty}$ is a sequence of positive numbers such that

    1.$a_1 \geq a_2 \geq .... \geq a_n \geq a_{n+1} \geq$..........and.

    2.$\lim_{n \to N} a_n = 0$ .Then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}$ is convergent.
11. Let (M,ρ_1) and (M,ρ_2) be a metric space and let f : $M_2 \to M_2$. Prove that f is continuous if and only if the inverse image of every open set is open.
12. Prove that the subset of A of R^1 is connected if and only if whenever a $\in$ A , b $\in$ A with a$\leq$b then c$\in$ A or any c such that a$\leq$c$\leq$b.
13. State and prove second fundamental theorem of calculus.

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