SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS APRIL-2022
SEMESTER - IV
20UMACT4007 - Vector Calculus and Fourier Transforms

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the directional derivative of xyz - $xy^2\ z^3$ at the point (1,2,-1) in the direction of the vector $\widehat{i}$ - $\widehat{j}$ - $3\widehat{k}$.
2. If $\overrightarrow{F}$ = yz $\overrightarrow{i}$ +zx $\overrightarrow{j}$ - xy $\overrightarrow{k}$, find $\int_C \overrightarrow{F}.\ d\overrightarrow{r}$ where C is given by x = t, y = t^2, z = t^3 from P(0,0,0) to Q(2,4,8).
3. Evaluate $\iiint_V \nabla \overrightarrow{F}$ dv if $\overrightarrow{F}$ =$x^2\overrightarrow{i}$ + $y^2\overrightarrow{j}$ + $z^2\overrightarrow{k}$ and if V is the volume of the region enclosed by the cube $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.
4. Find the Fourier sine transform of $\frac{1}{x}$.
5. Use Parseval's identity to show that $\int_0^\infty \frac{1}{(x^2+a^2)(x^2+b^2)}dx = \frac{\pi}{2ab(a+b)}$.
6. Find the angle between the normal to the surface xy - $z^2 = 0$ at the points (1,4,-2) and (-3,-3,3).
7. Define (i) Surface integral and (ii) Volume integral.
8. Find the Fourier sine and cosine transform of $f(x) = x$.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If $\overrightarrow{F}$ = $xy^2\overrightarrow{i}$ +2x^2 yz $\overrightarrow{j}$ - 3$yz^2\overrightarrow{k}$ find div $\overrightarrow{F}$ and curl $\overrightarrow{F}$. What are these values at the point (1,-1,1)?
10. Verify Green's theorem in the plane for $\oint (xy + y^2)\ dx + x^2\ dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.
11. Evaluate $\iint_S (x^3\overrightarrow{i} + y^3\overrightarrow{j} + z^3\overrightarrow{k}).\overrightarrow{n}\ dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$
12. Find the Fourier cosine transform of $\frac{1}{1+x^2}$ and hence find the Fourier sine transform of $\frac{x}{1+x^2}$.
13. Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1 & -|\ x\|x\ | < 1 \\ 0 & |\ x\ | > 1 \end{cases}$ Hence find the value of $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$.

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