

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.

B.Sc. END SEMESTER EXAMINATIONS APRIL-2022

SEMESTER - V

21UMACT5009 - Modern Algebra

Total Duration : 3 Hrs.

Total Marks : 60

Section A

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Prove : The subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G .
2. If ϕ is a homomorphism of G into \overrightarrow{G} with Kernel K . then prove that K is a normal subgroup of G .
3. If ϕ is a homomorphism of Ring R into R' then prove that $\phi(-a) = -\phi(a)$ for every $a \in R$.
4. If $\phi: R \rightarrow R'$ is a ring homomorphism of R into R' with Kernel U then Prove that R' is isomorphic to $\frac{R}{U}$.
5. Let R be a Euclidean ring, prove that any two elements a and b in R , have a greatest common divisor d and $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.
6. Let R be Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R , then prove that $d(a) \leq d(ab)$.
7. If $I(G)$ is the group of inner automorphism of G and Z is the center of G , then prove that $I(G) \cong \frac{G}{Z}$.
8. Prove that following
 - a. If G is a group such that $(a * b)^2 = a^2 * b^2$ for all $a, b \in G$ then prove that G is abelian group.
 - b. Justify: If H and K are subgroups of G , then $H \cap K$ is also subgroup of G .

Section B

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. State and Prove Lagrange's theorem.
10. State and prove Cayley's theorem.
11. Prove that any finite Integral domain is a field.
12. If R is a commutative ring with unit element and M is an ideal of R , then prove that M is a maximal ideal of R if and only if R/M is a field.
13. State and prove unique factorization theorem.
