SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS APRIL-2022
SEMESTER - V
08UMACT5012 - Graph Theory

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that a closed walk of odd length contains a cycle.
2. If G is a graph with p$\geq$3 vertices, and $\delta \geq \frac{p}{2}$,then prove that G is Hamiltonian.
3. Let G be a tree. Prove that any 2 vertices of G are joined by a unique path.
4. If any connected plane (p, q) graph(p$\geq$3) with r faces then prove that q$\geq \frac{3r}{2}$ and p$\leq$3 p-6
5. Prove that the following statements are equivalent for any graph.
   i. G is 2 colourable
   ii. G is bipartite
   iii. E very cycle of G has even length
6. Let $G = (p, q)$be a graph. Prove that $\delta \leq \frac{2q}{p} \leq \triangle$.
7. Define the following 1.Isomorphism of a graph
   2.K-critical graph
   3.Eulerian graph
   4.Thickness and crossing number of a graph
8. Prove that λ^4-3$\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that for any graph G, vertex connectivity$\leq$ line connectivity $\leq$ minimum degree of the graph. i.e. $K \leq \lambda \leq \delta$.
10. Prove that the following statements are equivalent for a connected graph
    1. G is Eulerian
    2. E very point of G has even degree
    3. The set of edges of G can be partitioned into cycles.
11. State and prove Hall's marriage theorem on Matching.

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12. If G is a connected graph having V , E and F as the set of vertices edges and faces respectively, then prove that $\mid V \mid - \mid E \mid + \mid F \mid$=2.
13. For any group G prove that δ=$\triangle$+1.

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