SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai — 600 044.

B.Sc. END SEMESTER EXAMINATIONS APRIL-2022

SEMESTER - VI

08UMACT6013 & UMA/CT/6013 - Linear Algebra

Total Duration : 3 Hrs. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that if $V_1, V_2, \ldots, V_n$ are in V then either they are linearly independent or some V_k is a linear combination of the preceding ones, $V_1, V_2, \ldots, V_{k-1}$.
2. Prove that if V is finite dimensional over F then any two bases of V have the same number of elements.
3. Prove that A(A(W))= W.
4. Derive the Cauchy – Schwarz inequality.
5. Prove that if V is finite dimensional over F and if T∈A(V) is singular, then there exists an s≠0 in A(V) such that ST=TS=0.
6. Prove that the element $\lambda \in$ F is a characteristic root of T∈A(V) if for some V≠0 in VT= λV.
7. Compute $\begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & -1 & -1 \end{pmatrix}$.
8. Let W⊂V is a invariant under T,then T induces a linear transformation $\bar{T}$ on $\frac{V}{W}$, defined by (V+W)$\bar{T}$=VT+W. If T satisfies the polynomial q(x)∈F⌈x⌉, then so does $\bar{T}$.If $P_1(x)$ is the minimal polynomial for $\bar{T}$ over F and if P(x) is for that T, then $\frac{p_1(x)}{p(x)}$.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If T is a homomorphism of U onto V with Kernal W, prove that V is isomorphic to U/W. Conversely if U is a vector space and W is a subspace of U, then prove that there is a homomorphism of U onto U/W.
10. Prove that Hom (V,W) is a vector space over F.
11. Let V be a finite dimensional inner product space then prove that V has an orthonormal set as a basis.
12. If $\lambda \in$ F is a characteristic root of T∈A(V) prove that λ is a root of the minimal polynomial of T.

contd...

13. If V is n dimensional over F and if T∈A(V) has the matrix m_1(T) in the basis V_1,V_2,.....V_n and the matrix m_2(T) in the basis W_1, W_2,.....W_n of V over F_1 then prove that there is an element $G \in F_n$ such that $m_2(T) = C\ m_1(T)C^{-1}$.

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