

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.

B.Sc. END SEMESTER EXAMINATIONS APRIL-2022

SEMESTER - VI

08UMACT6014 & UMA/CT/6014 - Complex Analysis

Total Duration : 3 Hrs.

Total Marks : 60

Section A

Answer any **SIX** questions ($6 \times 5 = 30$ Marks)

1. Prove that the following functions are nowhere differentiable.
 - i) $f(z) = \operatorname{Re} z$.
 - ii) $f(z) = e^x (\cos y - i \sin y)$.
2. Evaluate $\int_c \frac{z dz}{z^2 - 1}$ where c is the positively oriented circle $|z| = 2$.
3. Find the Taylor's Series to represent $\frac{z^2 - 1}{(z + 2)(z + 3)}$ in $|z| < 2$.
4. State and prove Rouché's theorem.
5. Determine the angle of rotation and scale factor at the point $z = 1 + i$ under the mapping $w = z^2$.
6. Prove that $f(z) = \begin{cases} \frac{z \operatorname{Re} z}{|z|} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ is continuous at $z = 0$ but not differentiable at $z = 0$.
7. State and prove Liouville's theorem.
8. Evaluate $\int_c \frac{z^2 dz}{(z - 2)(z + 3)}$ where c is the circle $|z| = 4$.

Section B

Answer any **THREE** questions ($3 \times 10 = 30$ Marks)

9. State and prove Cauchy-Riemann equations.
10. State and prove Cauchy's Integral formula.
11. Expand $\frac{-1}{(z - 1)(z - 2)}$ as a power series in z in the regions .
 - (i) $|z| < 1$
 - (ii) $1 < |z| < 2$

Contd...

12. Evaluate using

(i). Cauchy's Integral formula

(ii). Residue theorem

$$\int_c \frac{z+1}{z^2+2z+4} dz \text{ where } c \text{ is the circle } |z+1+i| = 2.$$

13. Prove that f be an analytic function defined in a region D . Let $z_0 \in D$. If $f'(z_0) \neq 0$ then f is conformal at z_0 .
