SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. - END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - V
20UMACT5009 - Modern Algebra

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. A nonempty subset H of the group G is a subgroup of G if and only if
   1. $a, b \in H$ implies that $ab \in H$. 2. $a \in H$ implies that $a - 1 \in H$.
2. Show that HK is a subgroup of G if and only if $HK = KH$.
3. Suppose G is a group, N a normal subgroup of G; define the mapping ϕ from G to G/ N by ϕ (x) = Nx for all x $\in$ G. Then ϕ is a homomorphism of G onto G / N.
4. Explain that let G be a group and ϕ an automorphism of G. If a $\in$ G is of order o(a) $>$ 0, then o(ϕ (a)) = o(a).
5. Prove that If ϕ is a homomorphism of R into R', then
   1. ϕ(0) = 0.
   2. ϕ(-a) = -ϕ(a) for every a $\in$ R.
6. Discuss that If R is a ring, then for all a, b $\in$ R
   1. a0 = 0a = 0.
   2. a (-b) = (-a) b = -(ab).
   3. (-a)(-b) = ab.
7. Show that If R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if R/ M is a field.
8. Prove that a Euclidean ring possesses a unit element.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that if H and K are finite subgroups of G of orders o(H) and o(K), respectively, then

$$\text{O(HK)} = \frac{O(H)O(K)}{O(H \cap K)}$$

Contd...

10. State and prove Cayley's theorem.
11. Explain that if 'p' is a prime number then Jp, the ring of integers mod p, is a field.
12. Prove that if 'U' is an ideal of the ring R, then R/U is a ring and is a homomorphic image of R.
13. Explain that every integral domain can be imbedded in a field.

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