SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - V
20UMACT5010 - Real Analysis

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Explain why a sequence of non-negative numbers cannot have a negative limit.
2. Show that the sequence $\left[\left(1+\frac{1}{n}\right)^n\right]_{n=1}^{\infty}$ is convergent.
3. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
4. State and prove comparison test for absolute convergence.
5. If E is any subset of metric space M, determine if $\bar{E}$ is closed.
6. Show that if $\langle M, \rho \rangle$ is a complete metric space and A is a closed subset of M, then $\langle A, \rho \rangle$ is also complete.
7. Show that if f has a derivative at every point of [a,b], then f' takes on every value between f'(a) and f'(b).
8. State and prove law of the mean.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. The set of all rational numbers in [0,1] is countable. Justify this statement.
10. $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers if and only if $\{S_n\}_{n=1}^{\infty}$ is convergent. Prove this claim.
11. Show that the real valued function f is continuous at 'a' if and only if $\lim_{n\to\infty} x_n = a$ implies $\lim_{n\to\infty} f(x_n) = f(a)$.
12. The subset A of M is totally bounded $\iff$ every sequence of points of A contains a Cauchy subsequence, when $\langle M, \rho \rangle$ is a metric space. Explain the validity of the statement.
13. State and prove fundamental theorem of calculus.

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