SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - V
20UMACT5012 - Graph Theory

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that for any graph G, q(G) $\geq$ p(G) - ω(G).
2. Prove that in a connected graph G there is an Eulerian trail iff the number of vertices of odd degree is either zero or two.
3. Show that if G is a Hamiltonian graph, then (G-S) $\leq$ |S|, for every non-empty subset S of V(G).
4. Show that a graph G is a tree iff every two vertices of G are connected by a unique path.
5. Show that, for a (p,q) graph G, the following statements are equivalent
   (i) G is a tree.
   (ii) G is connected and q=p-1.
   (iii) G is acyclic and q = p-1.
6. Explain and give example for the following
   (i) Planar Graph.
   (ii) Subdivision of edge.
   (iii) Edge contraction.
7. Prove that there exist a K-colouring of graph G iff V(G) can be partitioned into k subsets v_1, v_2,. . . ., v_k such that no two vertices of v_i, $i =$ 1,2,3,. . . k are adjacent.
8. Prove that for any Graph G, X(G) $\leq$ Δ(G)+1.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Show that if q $> \dfrac{p^2}{4}$,then every (p,q) – graph contains a triangle.
10. Justify that a nontrivial connected graph is Eulerian iff it has no vertex of odd degree.
11. Prove that a (p,q)- graph G is a bipartite graph iff it contains no odd cycles.

Contd...

12. State and prove Euler formula for planar graph. Also derive that if G is a plane (p,q)-graph in which every face is bounded by a cycle of length at least n then q$\leq \frac{n(p-2)}{n-2}$.

13. Prove that if G is a graph on p vertices, then

    (i) 2 $\sqrt{p} \leq \chi(\bar{G}) \leq$ p+1.

    (ii) p $\leq \chi(G)\chi(\bar{G}) \leq \frac{(p+1)^2}{4}$.

*****

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - V
20UMACT5012 - Graph Theory

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that for any graph G, q(G) $\geq$ p(G) - ω(G).
2. Prove that in a connected graph G there is an Eulerian trail iff the number of vertices of odd degree is either zero or two.
3. Show that if G is a Hamiltonian graph, then (G-S) $\leq$ |S|, for every non-empty subset S of V(G).
4. Show that a graph G is a tree iff every two vertices of G are connected by a unique path.
5. Show that, for a (p,q) graph G, the following statements are equivalent
   (i) G is a tree.
   (ii) G is connected and q=p-1.
   (iii) G is acyclic and q = p-1.
6. Explain and give example for the following
   (i) Planar Graph.
   (ii) Subdivision of edge.
   (iii) Edge contraction.
7. Prove that there exist a K-colouring of graph G iff V(G) can be partitioned into k subsets v_1, v_2,. . . ., v_k such that no two vertices of v_i, $i =$ 1,2,3,. . . k are adjacent.
8. Prove that for any Graph G, X(G) $\leq$ Δ(G)+1.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Show that if q $> \frac{p^2}{4}$,then every (p,q) – graph contains a triangle.
10. Justify that a nontrivial connected graph is Eulerian iff it has no vertex of odd degree.
11. Prove that a (p,q)- graph G is a bipartite graph iff it contains no odd cycles.

Contd...

12. State and prove Euler formula for planar graph. Also derive that if G is a plane (p,q)-graph in which every face is bounded by a cycle of length at least n then q$\leq \dfrac{n(p-2)}{n-2}$.

13. Prove that if G is a graph on p vertices, then

    (i) $2\sqrt{p} \leq \chi(\bar{G}) \leq$ p+1.

    (ii) p $\leq \chi(G)\chi(\bar{G}) \leq \dfrac{(p+1)^2}{4}$.

*****