SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - V
21UMACT5009 - Modern Algebra

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If G is a finite group and H is a subgroup of G , then prove that o(H) is a divisor of o(G).

2. If ϕ is a homomorphism of G into G' with kernel K, then prove that K is a normal subgroup of G.

3. If ϕ is a homomorphism of R into R' with kernel $I(\phi)$,then prove the following
   a) $I(\phi)$ is a subgroup of R under addition
   b) If a$\in I(\phi)$ and r$\in R$ then both ar and ra are in $I(\phi)$

4. If only ideals of the commutative ring R are (o) and R itself then prove that R is a field.

5. Let R be a Euclidean Ring. Prove that any 2 elements 'a' and 'b' in R have a greatest common divisor d and $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

6. Let R be a Euclidean ring. If that for $a, b, c \in R$, a/bc but $(a, b)=1$, then prove that a/c

7. If I $(G$) is the group of inner automorphism of G and Z is the center of G ,then prove that
   $I(G) \simeq \dfrac{G}{Z}$.

8. Prove that in a group $(G, *)$, G is a abelian $\iff (a * b)^2 = a^2 * b^2, a, b \in G$

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that the subgroup N of G is a normal subgroup of G iff every left coset of N in G is a right coset of N in G .

10. State and prove Cayley's theorem.

11. Prove that every finite integral domain is a field.

12. If R is a commutative ring with unit element and M is an ideal of R then prove that M is a maximal ideal of R iff R/M is a field.

13. Prove that every integral domain can be imbedded in a field.

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