SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - III
20UMAAT3003 - Mathematical Statistics - I

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. A Problem in Statistics is given to three students A, B and C whose chances of solving it are 1/2, 3/4 and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently ?

2. A bag contains 17 balls marked with the numbers 1 to 17. A ball is drawn and replaced; a second drawing is then made. What is the probability that
   i) The first number drawn is even and the second odd ?
   ii) The first number is odd and the second even ?

3. Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces, Construct a table giving the non-zero values of the probability mass function and draw the probability chart. Also find the distribution function of X.

4. A continuous random X has a p.d.f f(x) = $3x^2$, $0 \leq x \leq 1$.
   Find a and b such that
   i) P(X$\leq$a)=P(X $>$ a)
   ii) P(X $>$ b)=0.05

5. A coin is tossed until a head appears. What is the expectation of the number of tosses required?

6. Show that for p = 0.5, the binomial distribution has a maximum probability at * X = $\frac{1}{2}$ n , if n is even and at + X = $\frac{1}{2}$ (n-1) as well as - X = $\frac{1}{2}$ (n+1), if n is odd.

7. In a Poisson frequency distribution, frequency corresponding to 3 successes is 2/3 times frequency corresponding to 4 successes. Find the mean and Standard deviation of the distribution.

8. If X is uniformly distributed with mean 1 and Variance 4/3, find P(X$<$0).

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Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. i) State Bayes' Theorem.
   ii) The contents of urns I, II and III are as follows:
   1 White, 2 Black and 3 Red balls,
   2 White, 1 Black and 1 Red ball and
   4 White, 5 Black and 3 Red balls.
   One Urn is chosen at random and two balls drawn from it. They happen to be White and red. What is the Probability that they come from urns I, II or III.

10. The diameter, say X, of an electric cable, is assumed to be a continuous random variable with p.d.f. : f(x) = 6x(1-x), $0 \leq x \leq 1$
    i) Check that the above is a p.d.f.
    ii) Obtain an expression for the c.d.f. of X
    iii) Compute $P(X \leq 1/2 \mid 1/3 \leq X \leq 2/3)$, and
    iv) Determine the number k such that P(X<k) = P(X>k)

11. i) Give Chebychev's Inequality.
    ii) A Symmetric die is thrown 6000 times. Determine the lower bound for the Probability of getting 80 to 120 sixes.

12. Seven Coins are tossed and number of heads noted. The experiment is repeated 128 times and the following distribution is obtained :

No.of Heads	0	1	2	3	4	5	6	7	total
Frequencies	7	6	19	35	30	23	7	1	128

    Fit a binomial distribution assuming.
    i) The Coin is unbiased.
    ii) The nature of the coin is not known.
    iii) Probability of a head for 4 coins is 0.5 and for the remaining 3 coins is 0.45.

13. If X and Y are independent Gamma variates with parameters μ and ν respectively, Deduce that the variables U=X+Y, Z=X/(X+Y) are independent and that U is a $\gamma(\mu+\nu)$ variate and Z is a $\beta_1(\mu,\nu)$ variate.

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