SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER– 2022
SEMESTER – II
20UMACT2003 –Classical Algebra

Total Duration: 2 Hrs 30 Mins.   Total Marks: 60

Section A

Answer any *SIX* questions   (6 × 5 =30 Marks)

1. Show that $\left(\frac{1+x}{1-x}\right)^n = 1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{1.2}\left(\frac{2x}{1+x}\right)^2 + \ldots$

2. Sum to infinity the series $\frac{1}{1!} + \frac{1+5}{2!} + \frac{1+5^2+5^3}{3!} + \frac{1+5^2+5^3+5^4}{4!} + \cdots$

3. Solve the equation $x^5 - x^4 + 8x^2 - 9x - 15 = 0$ one root being $-\sqrt{3}$ and another 1+2i.

4. Show that $\frac{a^2}{x-\alpha} + \frac{b^2}{x-\beta} + \frac{c^2}{x-\gamma} - x + \delta = 0$ has only real roots if $a,b,c,\alpha,\beta,\gamma,\delta$ are all real.

5. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

6. a) Remove the fractional co efficient from the equation
$$x^3 - \frac{3}{2}x^2 + \frac{5}{18}x + \frac{1}{108} = 0$$
   b) Transform the equation $3x^3 + 4x^2 + 5x - 6 = 0$ into one in which the coefficient of x^3 is unity and all co efficient are integral.

7. Represent the matrix $\begin{bmatrix} 3 & -1 & 0 & 8 \\ 4 & 2 & -3 & -1 \\ 1 & 3 & -6 & 5 \\ -5 & 0 & -7 & -2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

8. Find the sum of the positive integers including unity which are less than 600 and prime to it.

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Section B

Part A

Answer any **THREE** questions (3× 10 =30 Marks)

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9. Sum the series

$$\frac{1}{1.2.3}\frac{1}{2} + \frac{1}{2.3.4}\cdot\frac{1}{2^2} + \frac{1}{3.4.5}\cdot\frac{1}{2^3} + \cdots.$$

10. Find the condition that the general biquadratic equation

$$ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$$

11. Solve the equation

$$6x^6 - 35x^5 + 56x^4 - 33 + 56x^2 + 35\,x - 6 = 0$$

12. Show that A= $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation

$A^2 - 4A - 5I = 0$. Hence determine its inverse.

13. State and prove Fermat's theorem.

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