SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER – 2022
SEMESTER – II

**20UMACT2004 – Integral Calculus and Fourier Series**

Total Duration: 2 Hrs 30 Mins. Total Marks: 60

**Section A**

Answer any ***SIX*** questions (6 × 5 =30 Marks)

1. Compute $\int x^3\, e^{2x}\ dx$.
2. Evaluate $\int e^x\, cos^2$ x dx
3. Evaluate $\int_0^a \int_0^x (x^2 + y^2)\, dy\, dx$.
4. Change the order of integration and evaluate $\int_0^a \int_y^a (x^2 + y^2)\, dy\, dx$.
5. Evaluate the integral $\int_0^\infty x^4\, e^{-x^4}\, dx$.
6. Evaluate $\int_0^{\pi/2} Sin^7\, \theta\, Cos^5 \theta\, d\theta$.
7. Express as a Fourier series the function

   $f(x) = \begin{cases} a & 0 < x < \pi \\ -a & \pi < x < 2\pi \end{cases}$

8. Find a Fourier Cosine series corresponding to the function $f(x) = x$ defined in the interval $(0, \pi)$.

**Section B**

**Part A**

Answer any ***THREE*** questions (3 × 10 =30 Marks)

9. Express $\boldsymbol{I_{m,n} = \int x^m (log\, x)^n\, dx}$ where m and n are positive integers. Hence or otherwise evaluate $\boldsymbol{\int x^4 (log\, x)^3\, dx}$.

Contd...

10. Evaluate $\iiint \frac{dx\ dy\ dz}{(x+y+z)^3}$ taken over the volume bounded by the plane $x = 0, y = 0, z = 0, x + y + z = 1$.

11. Express $\int_0^1 x^m(1-x^n)^p\ dx$ in terms of gamma function and evaluate the Integral $\int_0^1 x^5(1-x^3)^{10}\ dx$ .

12. If $f(x) = \begin{cases} -\pi & \text{in } -\pi < x < 0 \\ x & \text{in } 0 \le x \le \pi \end{cases}$ Prove that

$f(x) = \frac{-\pi}{4} - \frac{2}{\pi}(\cos x + \frac{\cos 3x}{3^2}) + \ldots) + 3 \sin x - \frac{\sin x}{2} + \ldots.$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$

13. Find a Fourier Cosines series corresponding to the function $f(x) = x$, defined in the interval $(0, \pi)$.

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