SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022
SEMESTER - IV
20UMACT4007 - Vector Calculus And Fourier Transforms

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the directional derivative of the function $x^2\,yz + 4xz^2$ at the point (1,-2,-1) in the direction of the vector $2\overrightarrow{i}$ - $\overrightarrow{j}$ - $2\overrightarrow{k}$.
2. If $\overrightarrow{F} = 3xy\overrightarrow{i}$ - $y^3\overrightarrow{j}$,Compute $\int_C \overrightarrow{F}.\mathrm{d}\overrightarrow{r}$ along y = $2x^2$ from (0,0) to (1,2).
3. Using Gauss divergence theorem,evaluate $\iint_S \overrightarrow{F}.\hat{n}$ ds where $\overrightarrow{F}$ = 4xz $\overrightarrow{i}$ - $y^2\overrightarrow{j} + yz\overrightarrow{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.
4. Derive a Fourier sine transform of the function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$
5. State and prove change of scale property of Fourier cosine transform.
6. If $\overrightarrow{F} = xyz\overrightarrow{i} + xyz^2\overrightarrow{j}$ + x^2yz $\overrightarrow{k}$, then find div(curl$\overrightarrow{F}$).
7. If $\overrightarrow{F}$ = ($2x^2$ - 3z)$\overrightarrow{i}$ - $2xy\overrightarrow{j}$ - $4x\overrightarrow{k}$, then find $\iiint_V(\nabla.\overrightarrow{F})$ dV, where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$.
8. State and prove shifting property of Fourier transform.

Section B

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If $\overrightarrow{r} = x\overrightarrow{i} + y\overrightarrow{j} + z\ \overrightarrow{k}$, Show that (i)$\nabla.r = \dfrac{\overrightarrow{r}}{r}(ii)\nabla.r^n = nr^{n-2}\overrightarrow{r}$.
10. Check Green's theorem for $\int_C (xy + y^2)dx + x^2\,dy$, Where C is the closed curve of the region bounded by the line $y = x$ and $y = x^2$.

Contd...

11. Verify Stoke's theorem for $\overrightarrow{F}$ =(x^2 - y^2)$\overrightarrow{i}$ + 2xy$\overrightarrow{j}$ taken over the rectangle bounded by $x = 0$, $x = a$, $y = 0$, $y = b$.

12. Find the Fourier transform of $f(x)= \begin{cases} 1 - x^2, \mid x \mid < 1 \\ 0, \mid x \mid \geq 1 \end{cases}$ and hence prove that $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2} ds = \frac{3\pi}{16}$.

13. Using transform methods, evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)}$ dx.

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