SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - II
20UMACT2003 - Classical Algebra

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Find the sum of series to infinity $\frac{1.4}{5.10} - \frac{1.4.7}{5.10.15} + \frac{1.4.7.10}{5.10.15.20} - ...$
2. Remove the fractional coefficients from the equation $x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{72} = 0$
3. Prove that a matrix is orthogonal iff its rows and columns are mutually orthogonal normal vectors
4. Find the smallest number with 18 divisors
5. Frame an equation with rational coefficients, one of whose roots is $\sqrt{5} + \sqrt{2}$.
6. Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$
7. Write the given matrix as $\begin{pmatrix} 2 & 1 & 4 \\ 8 & -1 & 3 \\ 3 & -5 & 0 \end{pmatrix}$ sum of symmetric and as skew symmetric matrix
8. Prove that the 5^{th} power of any integer N has the same unit digit as N.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Show that if a^r be the coefficient of x^n in the expansion of e^{e^x}, then $a^r = \frac{1}{r!}\left\{\frac{1^r}{1!} + \frac{2^r}{2!} + \frac{3^r}{3!} + ...\right\}$. Hence show that
   i. $\frac{1^3}{1!} + \frac{2^3}{2!} + \frac{3^3}{3!} + ... = 5e$
   ii. $\frac{1^4}{1!} + \frac{2^4}{2!} + \frac{3^4}{3!} + ... = 15e$

Contd...

10. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$

11. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$

12. Diagonalise the matrix $\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$

13. Show that if n is a prime number and $r < n$,
$(n-r)!(r-1)! + (-1)^{r-1} \equiv 0 (mod n)$

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