SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - II
20UMACT2004 - Integral Calculus and Fourier Series

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Evaluate $\int x^m (logx)^n dx$ (where m and n are positive integers)
2. Evaluate $\int e^2 x cos3x dx$
3. Evaluate $\iint (x^2 + y^2)\, dxdy$ over the region for which x , y are each ≥ 0 and x + y ≤ 1.
4. Evaluate $\int_0^1 x^m (log\frac{1}{x})^n dx$
5. Evaluate $\int_0^{\pi/2} \sqrt{tan\theta} d\theta$
6. Express f(x) = x ($-\pi < x < \pi$) as a Fourier series with period 2π.
7. Express f(x) = c-x where 0 < x < c as a half range cosine series with period 2c.
8. Find a sine series for f(x) = c in the range 0 to π.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Evaluate $\int sin^m x cos^n x dx$ (m, n being positive integers)
10. Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy\, dxdy$ and evaluate it.
11. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Contd...

12. Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty}(-1)^n \frac{cosnx}{n^2}$ in the interval (-$\pi \leq$ x $\leq \pi$).
    Deduce that
    (i) $\frac{1}{1^2}$ - $\frac{1}{2^2} + \frac{1}{3^2}$ ..... $= \frac{\pi^2}{12}$
    (ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}$ ..... $= \frac{\pi^2}{6}$
    (iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}$ ..... $= \frac{\pi^2}{8}$

13. Obtain a cosine series in the range 0 to π for $f(x) = \begin{cases} x & (0 \leq x < \pi/2) \\ -x & (\pi/2 \leq x < \pi) \end{cases}$

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