SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - IV

20UMACT4007 - Vector Calculus and Fourier Transforms

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that curl (grad ϕ) = 0 and div curl $\mathbf{F} = \mathbf{0}$
2. Prove that div ($r^n\bar{r}$) = (n+3) r^n and curl($r^n\bar{r}$)=0
3. Prove that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int(xdy - ydx)$ Hence, find the area of an ellipse.
4. Verify Green's theorem for integral I = $\int(x-2y)dx + xdy$ over the circle C $x^2 + y^2 = 1$.
5. Evaluate the integral I=$\int xdx + ydy + zdz$ where I is over a Circle C given by $x^2 + y^2 + z^2 = a^2$, z = 0 .
6. Show that the work done in moving a particle in a force field $\bar{F}$ = 3xy $\bar{i}$ + (x+y)$\bar{j}$ - z$\bar{k}$ along the curve x = t + 1, y = t - 1 and z = t^2 from (2,0,1) to (4,2,9) is -12.
7. Determine the Fourier cosine transform of e^{-x^2}
8. Using Parseval's identity, Show that $\int_0^\infty \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If $\bar{r}$ is the position vector of the point p(x,y,z), Prove
   i) div $\bar{r}$ = 3 ii) curl $\bar{r}$ = 0 iii) ∇r^n = n r^{n-2} $\bar{r}$
10. State Green's Theorem. Evaluate by Green's theorem for the integral I= $\int(xy + x^2)dx + (x^2 + y^2)dy$ over the square C formed by the lines x=-1, x=1, y=-1, y=1 in the XoY plane.

Contd...

11. State Gauss Divergence Theorem and Verify Gauss Divergence Theorem for $\bar{f} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ taken over the cube bounded by the planes x=0, x=1; y=0, y=1; z=0 and z=1

12. Determine the Inverse Fourier transform of $e^{-|s|}$ y

13. Define Convolution. State and Prove Convolution Theorem for Fourier Transform.

*****