

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)

Chromepet, Chennai — 600 044.

B.Sc. - END SEMESTER EXAMINATIONS APRIL-2023

SEMESTER - V

**08UMACT5012 - Graph Theory**

Total Duration : 2 Hrs 30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Prove that Every graph is an intersection graph.
2. Prove that if  $G$  is a graph in which the degree of every vertex is at least two then  $G$  contains a cycle.
3. Show that Every connected graph has a spanning tree.
4. Prove that  $K_5$  graph is non-planar.
5. Prove that if  $G$  is uniquely  $n$ - colourable, then  $\delta(G) \geq n-1$ .
6. Prove that any self complementary graphs has  $4n$  (or)  $4n+1$  points.
7. Prove that Every tree has a centre consisting of either one point or two adjacent points.
8. Define the following terms,  
(i) Complete graph. (ii) Walk. (iii) Connected graph.  
(iv) Eulerian graph. (v) Tree.

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. Prove that the maximum number of lines among all  $P$  point graphs with no triangles is  $\binom{\frac{p^2}{4}}{2}$
10. Prove that the following statements are equivalents for a connected graph  $G$   
(i)  $G$  is eulerian (ii) Every point of  $G$  has even degree  
(iii) The set of edges of  $G$  can be partitioned into cycles.
11. Let  $G$  be a  $(p,q)$  graph. Prove the following statements are equivalent  
(i)  $G$  is a tree (ii) Every two points of  $G$  are joined by a unique path  
(iii)  $G$  is connected and  $p=q+1$   
(iv)  $G$  is acyclic and  $p=q+1$

Contd...

12. Show that every polyhedron has atleast two faces with the same number of edges on the boundary.
13. Show that every planar graph is 5-colourable.

\*\*\*\*\*