20UMACT5009

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - V
20UMACT5009 - Modern Algebra

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If G is a finite group and H is a subgroup of G, then prove that o(H) is a divisor of o(G).
2. Let G be a group and ϕ is an automorphism of G, if $a \in G$ is of order o(a) then show that o(ϕ(a)) = o(a) .
3. Prove that a finite integral domain is a field.
4. Let R be a Euclidean ring d(a) = d(1) if and only if a is a unit in R.
5. Show that a subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G.
6. Define ϕ:$J(\sqrt{2}) \to J(\sqrt{2})$ by $\phi(m+n\sqrt{2}) = m-n\sqrt{2}$. Prove that ϕ is an onto homomorphism and find $I(\phi)$
7. If U is an ideal of R and $1 \in U$ then prove that R=U and if F is a field then its only ideals are element (0) and F itself.
8. Let R be the ring of all real values continuous functions on the closed unit interval. Let $M = \{f(x) \in R, [f(\frac{1}{2})] = 0\}$ Prove that M is a maximal ideal.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If H and K are finite subgroups of G of orders o(H) and o(K) respectively, then Prove that o(HK) = $\dfrac{o(H)o(K)}{o(H \cap K)}$.
10. State and Prove the fundamental theorem of homomorphism.
11. If R is a commutative ring with unit element and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field.
12. If U is an ideal of the ring, then show that R/U is a ring and is a homomorphism image of R.
13. State and Prove Unique factorization theorem.

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