SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - V
20UMACT5010 - Real Analysis

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that a monotonic increasing sequence which is bounded above is Convergent sequence.
2. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.
3. If F_1 and F_2 are two closed subsets of the metric space M, then show that, $F_1 \cup F_2$ is also closed in M
4. Show that, a closed subset of a complete metric space is also complete.
5. Verify Rolle's theorem for the function $f(x) = \sqrt{1-x^2}$, -1 $\leq$ x $\leq$ 1.
6. Show that, the sequence $\left(1+\frac{1}{n}\right)^n$ converges.
7. Show that, if f is a continuous function from a metric space M_1 into a metric space M_2 and if M_1 is connected then the range of f is also connected.
8. State and prove the law of mean.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Prove that, countable union of countable sets is also countable and hence show that, the set of rational numbers is countable.
10. Show that, a sequence $\{\mathbf{s}_n\}_{n=1}^{\infty}$ in R is convergent if and only if it is a Cauchy sequence.
11. Show that, if f is a function from a metric space M_1 into a metric space M_2, then f is continuous on M_1 if and only if $f^{-}1(G)$ is open in M_1 whenever G is open in M_2.
12. Let $<$ M, $\rho>$ be a complete metric space. If T is a contraction on M, then show that, there is one and only one point x in M such that Tx = x.
13. State and prove Fundamental theorem of calculus.

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