SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - VI
20UMACT6013 - Linear Algebra

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that if V is the internal direct sum of U_1,U_2,.....,U_n then V is isomorphic to the external direct sum of U_1,U_2,.....,U_n.
2. Prove that if v_1,v_2,.....,$v_n \in V$ are linearly independent, then every element in their linear span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 +$ ..... $+ \lambda_1 v_n$ with $\lambda_i \in F$
3. If V is finite dimensional over F and if u_1,u_2,.....,$u_m \in V$ are linearly independent, then we can find vectors u_{m+1},.....,u_{m+r} in V such that u_1,u_2,.....,u_m,u_{m+1},.....,u_{m+r} is a basis of V,
4. Show that if dim $_F$V $= m$ then dim $_F Hom(V, F) = m$
5. Discriminate $W^{\perp}$ is a subspace of V.
6. If V is a finite dimensional inner product space and if W is a subspace of V, then $V = W + W^{\perp}$. More particularly, V is the direct sum of W and $W^{\perp}$.
7. If V is a finite dimensional over F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.
8. If $W \subset V$ is invariant under T,then T induces a linear transformation $\hat{T}$ on V/W, defined by $(v + W)\hat{T} = vT + W$, if T satisfies the polynomial $q(x) \in F[x]$,then so does $\hat{T}$. If $p_1(x)$ is the minimal polynomial for $\hat{T}$ over F and if $p(x)$ is that for $p_1(x)|p(x)$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If v_1,v_2,.....,v_n in V have W as linear span and if v_1,v_2,.....,v_k are linearly independent , then we can find a subset of v_1,v_2,.....,v_n of the form v_1,v_2,.....,v_k,v_{i1},v_{i2},.....$v_{i,r}$ consisting of linear independent elements whose linear span is also W.

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10. If V is finite dimensional and if W is a subspace of V,then Prove that W is finite dimensional, dim $W \leq v$ and dim $V/W =$ dim V - dim W

11. State and prove Schwarz inequality

12. If λ_1,λ_2,.....,λ_k in F are distinct characteristic roots of $T \in A(V)$ and if v_1,v_2,.....,v_k are characteristic vectors of T belonging to λ_1,λ_2,.....,λ_k respectively then v_1,v_2,.....,v_k are linearly dependent over F.

13. If V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis v_1,v_2,.....,v_n and the matrix $m_2(T)$ in the basis w_1,w_2,.....,w_n of V over F, then there is an element $C \in F_n$ such that $m_2(T) = Cm_1(T)C^{-1}$. In fact , if S is the linear transformation of V defined by $v_1S = w_i$ for $i = 1, 2, 3, ...n$ then C can be chosen to be $m_1(S)$.

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