

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN  
(AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)  
Chromepet, Chennai — 600 044.

B.Sc.(Physics) - END SEMESTER EXAMINATIONS APRIL-2023

SEMESTER - III

**20UPHAT3003 - Allied Mathematics - I**

Total Duration : 2 Hrs 30 Mins.

Total Marks : 60

### Section B

Answer any **SIX** questions ( $6 \times 5 = 30$  Marks)

1. Show that  $\sqrt{x^2 + 16} - \sqrt{x^2 + 9} = \frac{7}{2x}$  nearly for sufficiently large value of  $x$

2. Prove that the matrix  $\frac{1}{2} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$  is orthogonal

3. Prove that  $\frac{\sin 7\theta}{\sin \theta} = 64\cos^6\theta - 80\cos^4\theta + 24\cos^2\theta - 1$

4. Using Newton's Interpolation formula, find the value of  $y$  when  $x=27$  from the following data

|   |      |      |      |    |      |
|---|------|------|------|----|------|
| x | 10   | 15   | 20   | 25 | 30   |
| y | 35.4 | 32.2 | 29.1 | 26 | 23.1 |

5. If  $\tan(\alpha + i\beta) = x + iy$  then prove that  $x^2 + y^2 + 2x\cos 2\alpha = 1$

6. Find Eigen value and Eigen vector of  $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

7. Show that  $\frac{1}{3!} + \frac{2}{5!} + \frac{3}{7!} + \frac{4}{9!} + \dots = \frac{1}{2e}$

8. Expand  $\cos 6\theta$  as a polynomial in  $\sin \theta$

### Section C

Answer any **THREE** questions ( $3 \times 10 = 30$  Marks)

9. Find the sum infinity of the series  $\frac{1}{24} - \frac{1.3}{24.32} + \frac{1.3.5}{24.32.40} - \dots$

10. Verify Cayley-Hamilton theorem for  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ . Hence find  $A^{-1}$

Contd...

11. Prove that  $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$

12. Using Lagrange's Interpolation formula, find  $y(10)$ , from the following table.

|   |    |    |    |    |
|---|----|----|----|----|
| x | 5  | 6  | 9  | 11 |
| y | 12 | 13 | 14 | 16 |

13. If  $\sin(A + iB) = x + iy$  prove that

$$\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$$

$$\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$$

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