SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL- 2023
SEMESTER - VI
08UMACT6013 - Linear Algebra

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If V is the internal direct sum of U_1,U_2...........U_n then prove that V is isomorphic to the external direct sum of U_1,............U_n
2. Prove that $A(A(w)) = W$ where $A(w)$ is annihilator of W.
3. If u,v$\in V$ then prove that $|\prec u, v \succ| \leq \|u\|\|v\|$.
4. If T,$S \in A(V)$ and if S is regular, then prove that T and STS^{-1} have the same minimal polynomials.
5. If V is n-dimensional vector space over F and if $T \in A(V)$ has all its characteristic roots in F then prove that T satisfies the polynomial of degree n over F.
6. Show that the vectors (0,1,1), (1,0,1) and (1,1,0) forms a basis for V_3(R).
7. Find the characteristics values and characteristics vectors of the matrix $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$
8. Using Gram-Schmitt process, obtain an orthogonal basis for the sub space of R^4 spanned by $x_1 =$ (1,0,1,0) $x_2 =$ (1,1,1,1) $x_3 =$ (-1,2,0,1)

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If V is a finite dimensional vector space, then prove that, it contains a finite set U_1,U_2..........U_n of linearly independent element and whose linear space in V
10. If V is finite-dimensional and if W is a subspace of V, then prove that $w \leq$ dim v and dim $v/w =$ dim v - dim w.
11. Prove that any finite dimensional inner product space V, has an orthonomal basis.

Contd...

12. If V is finite-dimensional vector space over F, then for $S,T \in A(V)$ prove the following:
a) r(ST) $\leq$ r(T)
b) r(TS) $\leq$ r(T)
c) r(ST) $=$ r(TS) $=$ r(T) for S - regular in $A(V)$
13. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

*****