SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Statistics) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - II

20USTAT2002 - Allied Mathematics - II

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Show that countable union of countable sets is countable.
2. Find $f'(x)$ for $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$ where 0 $\leq x < \infty$.
3. Compute L(sinh at).
4. Find the inverse Laplace transform of $\frac{s^3}{(s-4)^4}$
5. Examine the nature of the sequence $\left\{\left(1 + \frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$
6. State and prove the law of the mean theorem.
7. Find the Laplace transform of $f(t)$ if $f(t) = \begin{cases} e^{-t}, 0 \leq t \leq 4 \\ 0, 4 < t < \infty \end{cases}$
8. Solve $L^{-1}\left[\frac{s}{(s+1)^2 + 2^2}\right]$.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. Show that the set of all rational numbers in [0,1] is countable.
10. i. Predict for what values of p does the series $\frac{1}{1p} - \frac{1}{2p} + \frac{1}{3p} - \frac{1}{4p} + \ldots$ converge?

    ii. Prove that the series $\sum_{n=1}^{\infty}\left[\frac{1}{n(n+1)}\right]$ converges.
11. Determine the Taylor series about $x = 2$ for $f(x) = x^3 + 2x + 1, -\infty < x < \infty$. Prove that the Taylor series converges to $f(x)$ for every real x.
12. Find the Laplace transform of $e^{-t}\int_0^t \frac{sint}{t}dt$.
13. Evaluate $L^{-1}\left[\frac{4s+5}{(s-1)^2(s+2)}\right]$.

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