SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) - END SEMESTER EXAMINATIONS APRIL-2023
SEMESTER - V
21UMACT5009 - Modern Algebra

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If H is a non empty finite subset of a group G and H is closed under multiplication, then show that H is a subgroup of G.
2. Prove that a finite integral domain is a field.
3. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.
4. Let R be a$_n$ Euclidean ring and a, b $\in$ R. If b $\neq$ 0 is not a unit in R, then show that d(a) $<$ d(ab).
5. If ϕ is a homomorphism of G onto $\bar{G}$ with kernel K, then prove that K is a normal subgroup of G.
6. Prove that the homomorphism Φ of R into R' is an isomorphism if and only if I(Φ) = (0).
7. State and Prove Lagrange's theorem.
8. Prove that Every Euclidean Ring is a Principal Ideal domain.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. If H and K are finite subgroups of G of order o(H) and o(K) respectively, then prove that o(HK)= $\dfrac{o(H)o(K)}{o(H \cap K)}$.
10. State and Prove Cayley's Theorem.
11. Let R, R' be rings and ϕ a homomorphism of R onto R' with kernel U. Then prove that R' is isomorphic to R/U.
12. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.
13. In an Euclidean ring R, prove that any two elements a and b in R have a greatest common divisor d. More over d = $\lambda a + \mu b$ for some $\lambda, \mu \in R$.

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