SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
B.Sc.(Maths) END SEMESTER EXAMINATIONS April-2023
SEMESTER - VI
08UMACT6014 - Complex Analysis

Total Duration : 2 Hrs 30 Mins.   **Total Marks : 60**

Section B

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Prove that $f(z) = \begin{cases} \dfrac{zRez}{\mid z \mid} & if z \neq 0 \\ 0 & if z = 0 \end{cases}$ is continuous at z=0 but not differentiable at z=0.

2. Evaluate the integral $\int_c (x^2 - iy^2)dz$ where C is the parabola y=2x^2 from (1, 2) to (2, 8).

3. Expand cos z into a Taylor series about the point z= $\frac{\pi}{2}$ and determine the region of convergence.

4. Calculate the residue of $\dfrac{z+1}{z^2 - 2z}$ at its pole.

5. Discuss the transformation about w=sin z.

6. Prove that the following functions are nowhere differentiable
    (i) f(z)=Re z.
    (ii) f(z)=e^x (cosy-isiny).

7. Evaluate $\displaystyle\int_c \dfrac{z+2}{z}$ dz where C is the semicircle z=2e$^{i\theta}$ where $0 \leq \theta \leq \pi$.

8. Expand f(z)=sin z in a Taylor series about $z = \frac{\pi}{4}$ and determine the region of convergence of this series.

Section C

Answer any **THREE** questions $(3 \times 10 = 30$ Marks)

9. State and prove Cauchy-Riemann equation.

10. State and prove Cauchy's Integral formula.

Contd...

11. Expand $f(z) = \dfrac{z}{(z-1)(2-z)}$ in a Laurent's series valid for

      i) $|z| < 1$
      ii) $1 < |z| < 2$
      iii) $|z| > 2$
      iv) $|z\text{-}1| > 1$
      v) $0 < |z\text{-}2| < 1$

12. Evaluate

    $\displaystyle\int_c \dfrac{z+1}{z^2 + 2z + 4}\, dz$ where c is the circle $|z+1+i| = 2$ using

      (i) Cauchy's Integral formula.
      (ii) Residue theorem.

13. Discuss the transformation about $w = z^2$.

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