SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)

(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC) Chromepet, Chennai — 600 044.

B.Sc. END SEMESTER EXAMINATIONS NOVEMBER-2022 SEMESTER - I

20USTAT1001 - Allied Mathematics - I

Total Duration: 2 Hrs 30 Mins. Total Marks: 60

Section A

Answer any **SIX** questions $(6 \times 5 = 30 \text{ Marks})$

- 1. Find the coefficient of x^n in e^{e^x}
- 2. Find the n^{th} derivative of sin(ax + b)

3. If
$$\mathbf{x} = \mathbf{u}(1-\mathbf{v}) \ \mathbf{y} = \mathbf{u}\mathbf{v}(1-\mathbf{w}), \ \mathbf{z} = \mathbf{u}\mathbf{v}\mathbf{w}.$$
 Find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

- 4. Express $\cos 6\theta$ as a polynomial in
 - (1) $\cos \theta$ and
 - (2) $\sin \theta$
- 5. Find the reduction formula for $\int e^{ax}x^ndx$
- 6. Find the nth derivative of $\log (ax + b)$
- 7. Find the maxima and minima of the function $2x^3$ $3x^2$ 36x+10
- 8. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x dx$

Section B

Answer any **THREE** questions $(3 \times 10 = 30 \text{ Marks})$

9. Show that
$$1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots = \frac{e(e^2-1)}{2}$$

10. If
$$y = \sin(m \sin^{-1}x)$$
 show that $(1 - x^2)y_{n+2}$ $(2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$

- 11. Show that the maximum value of $x^2y^2z^2$ subject to the $x^2+y^2+z^2=a^2$ is $\frac{a^2}{3}$
- 12. Express $\frac{\sin 7\theta}{\sin \theta}$ as a polynomial in $\cos \theta$ and $\sin \theta$
- 13. If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$ prove that $I_n = \frac{n-1}{n} I_{n-2}$ hence evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x dx$
