20USTAT2002

SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS) (Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC) Chromepet, Chennai — 600 044. B.Sc. END SEMESTER EXAMINATIONS NOVEMBER – 2022 SEMESTER – II **20USTAT2002– Allied Mathematics – II**

Total Duration: 2 Hrs 30 Mins.

Total Marks: 60

Section A

Answer any *SIX* questions (6 × 5 = 30 Marks)

- 1. Which of the following statement is correct:
 - A: The inverse image of the intersection of two sets is the union of the inverse of images.
 - B. The inverse image of the union of two sets is the union of the inverse of images.
 - Justify your answer.
 - 2. If f is a continuous function on the closed bounded interval [a, b], f '(x) exists for all x in (a, b) and if there exists c in (a, b), then compute the value of f '(c).
 - 3. Show that
 - (i) $L(\cos at) = \frac{s}{s^2 + a^2}$

(ii)
$$L(\sin at) = \frac{a}{s^2 + a^2}$$

- 4. Determine the inverse Laplace transform of $\left(\frac{1}{(s+a)^2}\right)$.
- 5. Show that the sequence $\{10^7 / n\}_{n=1}^{\infty}$ has a limit 0.
- Show that the Taylor series about x = 0 for f(x) = sin x converges to sin x for every real x.
- 7. Compute Laplace transform of $\left[\frac{1-e^t}{t}\right]$.
- 8. Compute $L^{-1}\left(\frac{s}{(s+2)^2}\right)$.

Contd...

Section **B**

Part A

Answer any **THREE** questions $(3 \times 10 = 30 \text{ Marks})$

- 9. Show that the countable union of countable sets is countable and concluded that the set of all rational number is countable.
- If $\sum_{n=1}^{\infty} a_n$ is a divergent series of positive numbers, then show that there is a 10.

sequence $\{\mathcal{E}_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but for which $\sum_{n=1}^{\infty} \epsilon_n a_n$

still diverges.

- State and prove Rolle's theorem and examine the derivatives property 11. through an example.
- Predict the value of L($t e^{-t} sin t$). 12.
- Determine the inverse Laplace transform of 13.

$$\frac{1}{(s+1)(s^2+2s+2)}$$

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Section A

Answer any *SIX* questions $(6 \times 5 = 30 \text{ Marks})$

- 1. Which of the following statement is correct:
 - A: The inverse image of the intersection of two sets is the union of the inverse of images.
 - B. The inverse image of the union of two sets is the union of the inverse of images.
 Justify your answer.
- If f is a continuous function on the closed bounded interval [a, b], f '(x) exists for all x in (a, b) and if there exists c in (a, b), then compute the value of f '(c).
- 3. Show that
 - (i) $L(\cos at) = \frac{s}{s^2 + a^2}$

(ii)
$$L(\sin at) = \frac{a}{s^2 + a^2}$$

- 4. Determine the inverse Laplace transform of $\left(\frac{1}{(s+a)^2}\right)$.
- 5. Show that the sequence $\{10^7 / n\}_{n=1}^{\infty}$ has a limit 0.
- Show that the Taylor series about x = 0 for f(x) = sin x converges to sin x for every real x.

7. Compute Laplace transform of
$$\left[\frac{1-e^t}{t}\right]$$
.

8. Compute $L^{-1}\left(\frac{s}{(s+2)^2}\right)$.

Contd...

Section **B**

Part A

Answer any THREE questions (3 × 10 = 30 Marks)

- 9. Show that the countable union of countable sets is countable and concluded that the set of all rational number is countable.
- 10. If $\sum_{n=1}^{\infty} a_n$ is a divergent series of positive numbers, then show that there is

a sequence $\{\mathcal{E}_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but for $\sum_{n=1}^{\infty} 2^n = 0$

which
$$\sum_{n=1}^{n} a_n$$
 still diverges.

- 11. State and prove Rolle's theorem and examine the derivatives property through an example.
- 12. Predict the value of L(t e^{-t} sin t).
- 13. Determine the inverse Laplace transform of

$$\frac{1}{(s+1)(s^2+2s+2)}$$
