SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS) (Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC) Chromepet, Chennai — 600 044. B.Sc.(Statistics) END SEMESTER EXAMINATIONS APRIL-2023 SEMESTER - III **20USTCT3005 - Distribution Theory-II**

Total Duration : 2 Hrs 30 Mins.

Total Marks : 60

Section B

Answer any **SIX** questions $(6 \times 5 = 30 \text{ Marks})$

- 1. Derive the mean and variance of Exponential distribution.
- 2. State and prove the additive property of Cauchy distribution.
- 3. Show that the first two moments of standard Weibul distribution are $\Gamma\left(\frac{1}{c}+1\right)$ and $\Gamma\left(\frac{2}{c}+1\right)$
- 4. If X is a chi-square variate with n degrees of freedom, then prove that for large n, $\sqrt{2X}\sim (\sqrt{2n},1)$
- 5. For the 2 \times 2 contingency table

Prove that chi-square test of independence gives

$$\chi^{2} = \frac{N(ad - bc)^{2}}{((a + c)(b + d)(a + b)(c + d))} \text{ ,Where } \mathbb{N} = a + b + c + d$$

- 6. List at least three applications of t-distribution and also list the assumptions for Student's t-test.
- 7. Show that in $F(n_1, n_2)$ distribution, if we let $n_2 \to \infty$, then $\chi^2 = n_1 F$ follows χ^2 distribution with n_1 degrees of freedom.
- 8. Derive the pdf of the n^{th} order statistics.

Section C

Answer any **THREE** questions $(3 \times 10 = 30 \text{ Marks})$

- 9. Derive the moment generating function of the random variable X which follows the beta distribution.
- 10. Show that the odd moments of Logistic distribution are zero and the mean and variance of logistic distribution is 0 and $\frac{\pi^2}{3}$

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- 11. Show that the M.G.F. of the χ^2 distribution is $(1-2t)^{n/2}$ with |2t| < 1. Find its cdf and derive the first four moments.
- 12. Show that the limiting form of t-distribution tends to standard normal distribution.
- 13. Derive the pdf and cdf of distribution of range in order statistics.
