SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATIONS NOVEMBER – 2022
SEMESTER – I

17PAMCE1001 – Probability and Distributions

Total Duration: 2 Hrs 30 Mins. Total Marks: 60

Section A

Answer any ***SIX*** questions (6 × 5 =30 Marks)

1. Suppose that an urn contains b white and c black balls, b -f $c = N$. A ball is drawn at random, and before drawing the next ball, 5 + 1 balls of the same color are added to the urn. The procedure is repeated n times. Let X be the number of white balls drawn in n draws, $X = 0, 1, 2, \dots, n$. Determine the PMF of X.

2. Let $X \sim C(\mu_1, \vartheta_1)$ and $Y \sim C(\mu_2, \vartheta_2)$ be independent Random variables. Then Prove that $X+Y$ is a $C(\mu_1 + \mu_2, \vartheta_1 + \vartheta_2)$ Random variable.

3. An urn contains three red and two green balls. A random sample of two balls is drawn
   (a) with replacement, and
   (b) with out replacement. Let $X = 0$ if

   the first ball drawn is green, = 1 if the first ball drawn is red, and let $Y = 0$ if the second ball drawn is green, = 1 if the second ball drawn is red. The joint PMF of (X, Y) is given in the following tables:

   (a) With replacement

Y \ X	0	1	
0	$\frac{4}{25}$	$\frac{6}{25}$	$\frac{2}{5}$
1	$\frac{6}{25}$	$\frac{9}{25}$	$\frac{3}{5}$
	$\frac{2}{5}$	$\frac{3}{5}$	1

   (b) Without replacement

Y \ X	0	1	
0	$\frac{2}{20}$	$\frac{6}{20}$	$\frac{2}{5}$
1	$\frac{6}{20}$	$\frac{6}{20}$	$\frac{3}{5}$
	$\frac{2}{5}$	$\frac{3}{5}$	1

   Determine its conditional PMFs and the conditional expectations.

4. Let (X, Y) be jointly distributed with density function

   $$f(x, y) = \begin{cases} x + y, & 0 < x < 1, \quad 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Infer its Covariance and Correlation coefficient.

5. Derive MGF, Mean and Variance of Non-central Chi-Square distribution.

6. Ascertain: $(n-1)S^2/\sigma^2$ is $\chi^2(n-1)$
7. Prove that $X_n \xrightarrow{P} X$ *implies* $X_n \xrightarrow{L} X$
8. Establish an example to show that the convergence in distribution does not imply convergence of moments.

Section B
Part A

Answer any ***TWO*** questions (2 × 10 =20 Marks)

9. Define Gamma Distribution. Find MGF, Mean and Variance of Gamma Distribution. Illustrate few graphs of its PDFs.

10. Suppose that *r* balls are drawn one at a time without replacement from a bag containing ***n*** white and ***m*** black balls. Evaluate its Variance and Covariance.

11. Define Bivariate Normal Distribution. Prove that the functions defined in its definitions are a Joint PDF. Also Establish that the marginal PDFs of *X* and *Y* are, respectively, $N(\mu_1, \sigma_1^2)$ & $N(\mu_2, \sigma_2^2)$ and ρ is the correlation coefficient between *X* and *Y.*

12. Let $X_1, X_2, \ldots X_n$ be the $N(\mu, \sigma^2)$ Random Variables. Then Prove that $\bar{X}$ and $X_1 - \bar{X},\ X_2 - \bar{X}, \ldots, X_n - \bar{X}$ are independent.

Part B

Compulsory Question (1 × 10 = 10 Marks)

13. State and Deduce Lindeberg-Levy Central Limit Theorem along with its Converse part.

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