SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS NOVEMBER - 2022
SEMESTER - IV
20PAMCT4010 - Functional Analysis

Total Duration : 2 Hrs 30 Mins. **Total Marks : 60**

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If N a normed linear space, then show that the closed unit sphere S* in N* is a compact Hausdorff space in the weak* topology
2. State and prove open mapping theorem.
3. If T is an operator on H for which (Tx,x) = 0 for all x then prove that T = 0.
4. Prove that the boundary of set of all singular elements S is a subset of Z
5. Prove that following in Banach Algebra
   a. If r is an element of R then 1 – r left regular
   b. If r is an element of R then 1 – r regular
6. State and prove closed graph theorem.
7. Prove that the adjoint operator T →T* on B(H) has the following properties
   $(T_1 + T_2)^*= T_1^* + T_2^*$
   $(\alpha T)^*= \bar{\alpha} T^*$
   $(T_1T_2)^*= T_2^* T_1^*$
   T** = T
   ‖ T* ‖ = $\|T\|$
8. Prove that in Banach Algebra σ(x) is nonempty.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. State and prove Hahn- Banach theorem.
10. Let H be a Hilbert space, and let $\{e_i\}$ be an orthonormal set in H. then prove that the following conditions are equivalent to one another
    1) $\{e_i\}$is complete
    2) $x\perp \{e_i\} \Rightarrow$ x = 0
    3) if x is an arbitrary vector in H then x = $\sum (x, e_i)e_i$
    4) if x is an arbitrary vector in H then $\|x\|^2 = \sum | (x, e_i) |^2$

Contd...

11. If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other then prove that $N_1 + N_2$ and $N_1 N_2$ are normal Also prove that if N is a normal operator on H then $\|N^2\| = \|N\|^2$.

12. Prove that the spectral radius r(x) $= lim \ \|x^n\|^{1/n}$.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Gelfand- Neumark theorem.

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