SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS NOVEMBER - 2022
SEMESTER - II
20PAMET2002 - Mathematical Statistics

Total Duration : 2 Hrs 30 Mins.   Total Marks : 60

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. Let (X , Y) be jointly distributed with Probability Density Function
   f_θ(x,y) = exp[-(x/θ + θy)], x > 0, y > 0. Find the Fisher information in a sample
   of n pairs.

2. State and Prove Rao - Blackwell Theorem.

3. Define i) family of random sets
         ii) lower confidence bound for θ
         iii) uniformly most accurate (UMA) lower confidence bound
         iv) UMA family of confidence sets

4. Let $T(x)$ be maximal invariant with respect to G. Then Prove that ϕ is invariant
   under Gif and only if ϕ is a function of T.

5. Explain i). unbiased Size α test
         ii). an UMP unbiased size α test.
         iii). α - similar test
         iv). UMP α - similar test

6. Establish an example due to Stein and Rubin to show that GLR test does not
   perform well.

7. A die is rolled 120 times with the following results:

Result	1	2	3	4	5	6
Frequency:	20	30	20	25	15	10

   Test the hypothesis that the die is fair at level $\alpha = 0.05$?

Contd...

8. Three sections of the same elementary statistics course were taught by three instructors, I, II, and III. The final grades of students were recorded as follows:

I	II	III
95	88	68
33	78	79
48	91	91
76	51	71
89	85	87
82	77	68
60	31	79
77	62	16
	96	35
	81	

Test the hypothesis that the average grades given by the three instructors are the same at level $\alpha = 0.05$?

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks$)$

9. Let $X_1, X_2, \dots , X_n$ be a sample from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Then infer its Maximum Likelihood Estimators.

10. Establish the proof of Neyman-Pearson Fundamental Lemma.

11. i) The following data were obtained from a table of random numbers of normal distribution with mean 0 and variance 1.

0.464	0.137	2.455	-0.323	-0.068
0.906	-0.513	-0.525	0.595	0.881
-0.482	1.678	-0.057	-1.229	-0.486
-1.787	-0.261	1.237	1.046	-0.508

Test the null hypothesis that the Distribution Function 'F' from which the data came is normal with mean 0 and variance 1.

Contd...

ii) In a 72-hour period on a long holiday weekend, there was a total of 306 fatal automobile accidents. The data are as follows:

Number of Fatal Accidents per Hour	Number of Hours
0 or 1	4
2	10
3	15
4	12
5	12
6	6
7	6
8 or more	7

Test the hypothesis that the number of accidents per hour is a Poisson Random Variable.

12. Discuss about Testing Significance of Regression.

Part B

Compulsory question $(1 \times 10 = 10$ Marks$)$

13. State and Deduce Cramer-Rao inequality.

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