SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN (AUTONOMOUS) (Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC) Chromepet, Chennai — 600 044. M.Sc. - END SEMESTER EXAMINATIONS NOVEMBER - 2022 SEMESTER - 1

22PAMET1001 - Probability and Distributions

Total Duration : 2 Hrs 30 Mins.

Total Marks : 60

Section A

Answer any **SIX** questions $(6 \times 5 = 30 \text{ Marks})$

- 1. Find the mean and variance of Poisson distribution.
- 2. Let (X, Y) be jointly distributed with probability density function, f (x, y) = 2, 0 < x < y < 1 and f (x, y) = 0 otherwise. Find the marginal probability distribution functions of X and Y.
- 3. Find the moment generating function of bivariate normal distribution.
- 4. Find the mean and variance of F distribution.
- 5. Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{2} X$. Then show that, $EX_n \rightarrow EX$ and $EX_n^2 \rightarrow EX^2$ as $n \rightarrow \infty$.
- 6. Find the moment generating function of Uniform distribution.
- 7. State and prove additive property of Chi-square distribution.
- 8. Show that, $E(h(X)) = E\{E\{h(X) \mid Y\}\}$ if E(h(X)) exists.

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20 \text{ Marks})$

- 9. Find the moment generating function of Binomial distribution and also find mean and variance.
- 10. Let X_1, X_2 be independent random variables with common density given by

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & otherwise \end{cases}$$
 Let $Y_1 = X_1 + X_2, Y_2 = X_1 - X_2.$

Find joint pdf, marginal pdf of Y_1 and Y_2 .

- 11. Let $X = (X_1, X_2, ..., X_n)$ be an n-dimensional random variable with a normal distribution. Let $Y_1, Y_2, ..., Y_k, k \leq n$, be linear function of X_j (j=1, 2, ...,n). Then, show that, $(Y_1, Y_2, ..., Y_n)$ also has a multivariate normal distribution.
- 12. Find the moment generating function of t-distribution and find its mean and variance.

Part B

Compulsory question $(1 \times 10 = 10 \text{ Marks})$

13. State and prove Lindberg-Levy form of central limit theorem.
