SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS NOVEMBER - 2022
SEMESTER - I
20PAMCT1001 - Algebra I

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. If O(G) = p^n where p is a prime number, then prove that Z(G) $\neq$ (e)
2. Define n(k) and show that n(k)= 1 + p + ... + p^{k-1}
3. Define G(s) and prove that that for any two isomorphic abelian group G and G' , for every integer s, G(s) and G'(s) are isomorphic.
4. Prove that G is solvable iff $G^{(k)}$ = (e) for some integer k.
5. Define transpose of a matrix and show that for all A,B $\in$ Fn, (AB)'= B'A'
6. Define Unitary and Show that if (vT, vT) = (v,v) for all v $\in$ V then T is unitary.
7. Show that for every prime number p and every integer m there exists a field having p^m elements.
8. Define adjoint and prove that adjoint in Q satisfies
   (i) $x^{**} = x^*$
   (ii) $(\sigma x + \gamma y)^* = \sigma x^* + \gamma y^*$
   (iii) $(xy)^* = y^* x^*$

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. State and Prove Sylow's theorem.
10. Show that for a Euclidean ring R any finitely generated R-module, Mi is the direct sum of a finite number of cyclic submodules.
11. If T $\in$ A(V) then prove that T^* $\in$ A(V). Also,
    (i) $(T^*)^* = T$
    (ii) $(S + T)^* = S^* + T^*$
    (iii) $(\lambda S)^* = \bar{\lambda} S^*$
    (iv) $(ST)^* = T^*S^*$

Contd...

12. Write the statement of Weddurburn thorem and prove it.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. State and Prove Frobenius theorem with illustration.

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