SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with 'A+' Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. - END SEMESTER EXAMINATIONS NOVEMBER - 2022
SEMESTER - I

20PAMCT1002 - Real Analysis

Total Duration : 2 Hrs 30 Mins. Total Marks : 60

Section A

Answer any **SIX** questions $(6 \times 5 = 30$ Marks)

1. (i) Define σ - algebra.
   (ii) Define Lebesgue outer measure for subsets of real numbers.
2. Explain Uniform Convergence.
3. If X is a complete metric space, and if φ is a contraction of X into X , then there exists one and only $x \in X$ such that $\varphi(x) = x$.
4. Suppose $\sum c$ converges, put
   $f(x)=\sum_{n=0}^{\infty} c_n x^n$ (-1 $< x <$1)
   Then prove that
   $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$
5. State Fatou's Lemma and give an example
6. If K is compact, if $f_n \in \mathcal{C}(K)$and if $\{f_n\}$ is point wise bounded and equicontinuous on K, prove that $\{f_n\}$ is uniformly bounded on K.
7. State inverse function theorem.
8. If for some x there are constants
   $\delta > 0$ and $M < \infty$ such that
   $| f(x+t) - f(x) | \leq M | t |$
   For all
   $t \in (-\delta, \delta)$ then prove that $\lim_{N \to \infty} s_N(f : x) = f(x)$.

Contd...

Section B

Part A

Answer any **TWO** questions $(2 \times 10 = 20$ Marks)

9. (a) Let $\{E_i\}$ be a sequence of measurable sets . Then Prove that
   (i) If $E_1 \subseteq E_2 \subseteq$ ..., we have $m(\lim E_i) = \lim m(e_i)$
   (ii) If $E_1 \supseteq E_2 \supseteq$..., and $m(E_i) < \infty$ for each i, then we have $m(\lim E_i) = \lim m(E_i)$

   (b)Prove that every interval is measurable.

10. If f is Riemann integrable and bounded over the finite interval [a,b] then Prove that f is integrable and $R\int_a^b f dx = \int_a^b f dx$

11. State and prove the Stone – Weierstrass theorem.

12. State and prove the Implicit function theorem.

Part B

Compulsory question $(1 \times 10 = 10$ Marks)

13. State and prove Bessel's inequality.

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