SHRIMATHI DEVKUNVAR NANALAL BHATT VAISHNAV COLLEGE FOR WOMEN
(AUTONOMOUS)
(Affiliated to the University of Madras and Re-accredited with A+ Grade by NAAC)
Chromepet, Chennai — 600 044.
M.Sc. END SEMESTER EXAMINATIONS NOVEMBER – 2022
SEMESTER – II

20PAMCT2004 - Algebra II

Total Duration: 2 Hrs 30 Mins. Total Marks: 60

Section A

Answer any ***SIX*** questions (6 × 5 =30 Marks)

1. If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

2. If a, b in K are algebraic over F then show that $a \pm b$, ab, and a/b (if $b \neq 0$) are all algebraic over F.

3. If $p(x) \in F[x]$ and if K is an extension of F, then prove that for any element $b \in K, p(x) = (x - b)q(x) + p(b)$ where $q(x) \in K[x]$ and where $\deg q(x) = \deg p(x) - 1$.

4. For any $f(x), g(x) \in F[x]$ and any $\alpha \in F$, prove that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

5. Show that the fixed field of G is a subfield of K.

6. If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \cdots \alpha_m T^m$ where $\alpha_i \in$ F, is invertible if $\alpha_0 \neq 0$.

7. If u$\in V_1$ is such that $uT^{n_1-k} = 0, where\ 0 < k \leq n_1$, then prove that u = $u_0 T^k$ for some $u_0 \in V_1$

Contd...

8. If T in $A_F(V)$ has as minimal polynomial $p(x) = q(x)^e$, $q(x)$ is a monic, irreducible polynomial in $F[x]$, then show that a basis of V over F can be found in which the matrix of T is of the form

$$\begin{pmatrix} C(q(x)^{e_1}) & & \\ & C(q(x)^{e_2}) \ddots & \\ & & C(q(x)^{e_r}) \end{pmatrix}$$

Where $e=e_1 \geq e_2 \geq \ldots\ldots\ldots\ldots \geq e_r$

Section B

Part A

Answer any ***TWO*** questions (2 × 10 = 20 Marks)

9. Prove that the number e is transcendental.

10. If F is of characteristic 0 and if a, b, are algebraic over F, then prove that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

11. If K is afinite extension of F, then show that G(K, F) is afinite group and its order, o(G(K, F)) satisfies o(G(K, F)) ≤ [K:F].

12. If T∈A(V) has all its characteristic'roots in F, then show that there is a basis of V in which the matrix of T is triangular.

Part B

Compulsory Question (1 × 10 = 10 Marks)

13. For each i = 1,2,….k, $V_i \neq (0)$ and V = $V_1 \oplus V_2 \oplus \ldots\ldots..\oplus V_k$. Prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

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20PAMCT2004

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